XIV. *On a Determination of the Mean Density of the Earth and the Gravitation Constant by means of the Common Balance.*

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Received May 13,—Read June 4, 1891.

[Plates 13–19.]

I. Account of Apparatus and Method.

In a paper printed in the 'Proceedings of the Royal Society,' No. 190, 1878 (vol. 28, pp. 2–35), I gave an account of some experiments undertaken in order to test the possibility of using the Common Balance in place of the Torsion Balance in the Cavendish Experiment. The success obtained seemed to justify the intention expressed in that paper to continue the work, using a large bullion balance, instead of the chemical balance with which the preliminary experiments were made.

As I have had the honour to obtain grants from the Royal Society for the construction of the necessary apparatus, I have been able to carry out the experiment on the larger scale which appeared likely to render the method more satisfactory, and this paper contains an account of the results obtained.

At the time I was making the preliminary experiments the late Professor v. Jolly was already employing the balance for gravitation investigations ('Wiedemann's Annalen,' vol. 5, p. 112), though I was not aware of the fact. Later he published an account ('Wied. Ann.,' vol. 14, p. 331) of a determination of the Mean Density of the Earth by the use of the Balance. Still more recently Drs. Kœnig and Richarz have devised a method of using the balance for the same purpose ('Nature,' vol. 31, pp. 260 and 475), and I believe that their work is still in progress. It might appear useless to add another to the list of determinations, especially when, as Mr. Boys has recently shown, the torsion balance may be used for the experiment with an accuracy quite unattainable by the common balance. But I think that in the case of such a constant as that of gravitation, where the results have hardly as yet begun to close in on any definite value, and where, indeed, we are hardly assured of the constancy itself, it is important to have as many determinations as possible made by different methods and different instruments, until all the sources of discrepancy are traced and the results agree.

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The apparatus for the experiments described in this paper was first set up in the Cavendish Laboratory at Cambridge through the kindness of Professor CLERK MAXWELL. After spending some months in working at the experiment, but without much success beyond the detection of some sources of error, I left Cambridge, and ultimately the apparatus was again set up at the Mason College, Birmingham. The difficulties in carrying out the work with any approach to exactness have been far greater than were anticipated, and many times work has been begun and results have been obtained, but examination has shown them to be affected by large errors which could be traced and eliminated by further improvements in the apparatus.

At the beginning of 1890, however, the apparatus was brought into fair working order, and during the course of the year I made a number of experiments with the results recorded in this paper.

The Principle of the Experiment.

The object of the experiment, in common with all of its class, may be regarded, primarily, as the determination of the attraction of one known mass M on another known mass M′ a known distance d away from it. The law of universal gravitation states that when the masses are spheres with centres d apart this attraction is GMM'/d^2, G being a constant—the gravitation constant—the same for all masses. Astronomical observations fully justify the law as far as M'/d^2 is concerned. They do not, however, give the value of G, but only that of the product GM for various members of the solar system.

To determine G we must measure GMM'/d^2 in some case in which both M and M′ are known, whether they be a mountain and a plumb bob, as in MASKELYNE's experiment, the surface strata and a pendulum bob, as in AIRY's experiment, or two spheres of known mass and dimensions, as in all the various forms of CAVENDISH's experiment.

Knowing the gravitation constant G, we may at once find the mean density of the earth Δ. For if V be the volume of the earth—regarded as a sphere of radius R—the weight of any mass M′, being the attraction of the earth on it, is

$$GV\Delta M'/R^2.$$

But if g is the acceleration of gravity the weight is also expressible as $M'g$.

Equating these we get

$$\Delta = gR^2/GV.$$

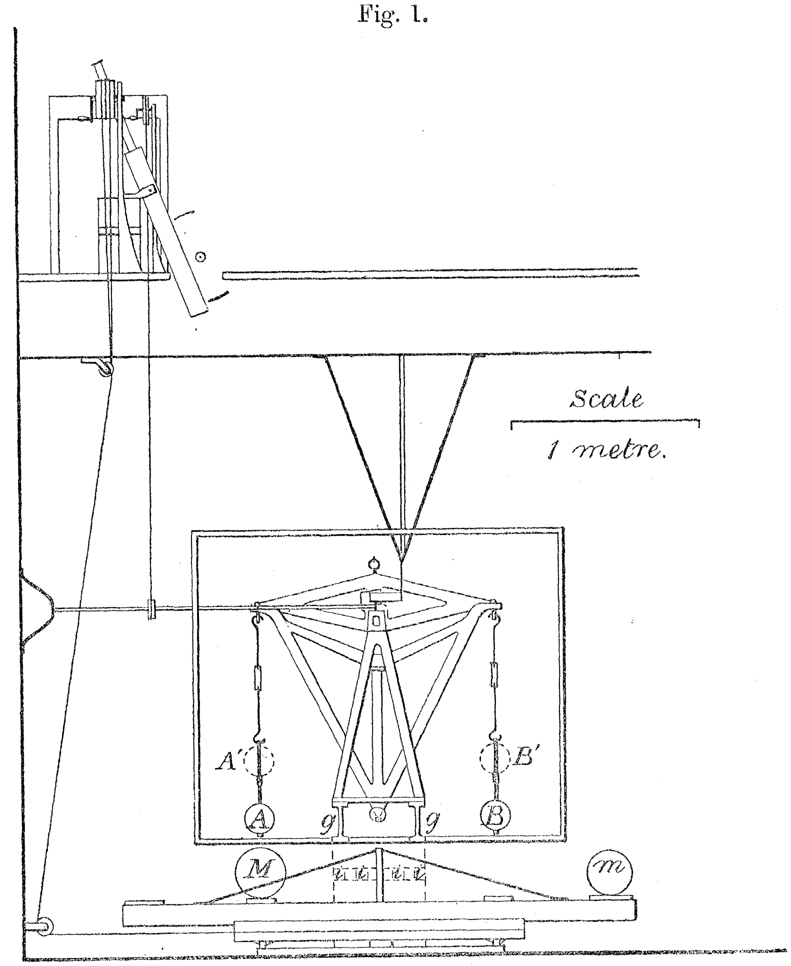
Method of Using the Common Balance.

In using the common balance to find the attraction between two masses, perhaps the most direct mode of proceeding would consist in suspending a mass from one arm of a balance by a long wire, and counterpoising it in the other pan. Then bringing under it a known mass, its weight would be slightly increased by the attraction of this mass. The increase would be the quantity sought if the attracting mass had no appreciable effect before its introduction beneath the hanging mass, and if, when beneath it, the effect on the balance could be neglected. This is very nearly the principle of the method used by VON JOLLY, and it is that of the method used in the preliminary experiments referred to above, in which a mass of 453 grms. of lead was hung from one arm of a chemical balance (about 40 centims. beam) by a wire 1·8 metres long, and was attracted by a mass of 154 kilogrms. of lead. But the attraction to be measured was exceedingly small, rather less than 0·01 milligrm., and it therefore appeared advisable to use a much larger balance with a larger hanging mass so that the attraction might be made comparable with the weight of exactly determined riders. Other anticipations as to proportionate increase of sensibility and diminution of effect of air currents, have hardly been justified in the way I expected, though, by the ultimate form of the apparatus, they have, I think, been more than realised.

With increase in the length of beam, a differential method became applicable, by means of which the attraction of the mass on the beam was eliminated, and the necessity for prolonging the case to allow of a long suspending wire was removed. This will be seen from a consideration of fig. 1. Let AB represent equal masses suspended from the two arms of the balance, and let M be the attracting mass put first under A, the position of the beam being noted. If M is then placed under B its attraction is not only taken away from A but added to B, so that the tilting of the beam is that due to nearly double the attraction to be measured. Of course there are what we may term *cross-attractions*, in the first position, of M on B, and in the second position, of M on A, but these may be allowed for in the calculations. We cannot give any mathematical expression for the attraction of M on the beam and suspending wires, owing to their irregularity of shape. But this attraction is eliminated if a second experiment is made in which A and B are raised equal known distances to A′ and B′. For the *difference* between the two increments of weight on the right, is due solely to the alteration of the positions of A and B relative to M, the attraction on the beam remaining the same in each. From the observed effect of a known alteration of distance the attraction at any distance can be found.

This is, shortly, the method adopted. The arrangement was ultimately complicated by the addition of a second mass m. Originally the mass M was alone on a turn-table which revolved about a vertical axis immediately under the central knife-edge of the balance. And some experiments which I made led me to suppose that mere change of position of the mass did not affect the level of the balance. However, after a complete determination in 1888 of the mean density, when I supposed that the work was finished, an examination of the results showed some curious anomalies, which I could only ascribe to a tilting of the whole floor on the displacement of the mass. Making new tests as to the effect of removal of the mass, I found that the previous tests had been quite wrong in principle, and that there was a very appreciable effect quite visible in the telescope when the masses A and B were removed, and M was removed from one side to the other, the slope of the floor changing by an angle comparable with a third of a second. If this had been absolutely constant in amount, the differential method would have eliminated it; but, probably, it varied slightly in successive motions of the turn-table, and the results showed that

Fig. 1.



Elevation of balance room and observing room. The front of the case is removed, and the front pillar is not shown. The pointer and mirrors are at the back.

there was also a secular change, the amount of tilt gradually increasing. This secular change was probably due to increasing rigidity of the floor, so that it tilted over bodily, moving the supports of the balance with it, an increase partly due, perhaps, to the pressure of the building, which had only been erected ten or twelve years, but chiefly, I think, to a gas engine recently erected next door. When this was doing heavy work, the vibrations were very plainly felt, and no doubt they greatly aided the floor in "settling down." A second balancing mass m was therefore added, half as great as M, and on the opposite side of the turn-table, but twice as far from the axis. The resultant pressure was now always through the axis, and I could detect no tilting of the floor when the turn-table was moved. Of course the balancing mass acted somewhat to reduce the effect of the larger attracting mass, but in a calculable ratio.

Finally, in order to eliminate or reduce the effect of any want of symmetry in the moving parts or in the masses, a second set of experiments was made with all the masses turned over and moved from left to right, and the mean of the first and second set was taken.

I now proceed to a detailed description of the various parts of the apparatus and the mode of experiment.

The Balance Room.—The balance room is in the basement of the Mason College, immediately under my room, and about 20 metres from the street. On one side were three windows looking on to a small courtyard, entirely surrounded by high buildings, but the windows have been bricked up. On the two adjacent sides are two other rooms, and on the opposite side a closely fitting door opening on a short corridor with doors at each end. There is no chimney in the room, and only an opening in the ceiling through which the balance was observed from the room above. The floor is of brick, resting on earth, and is very firmly laid.

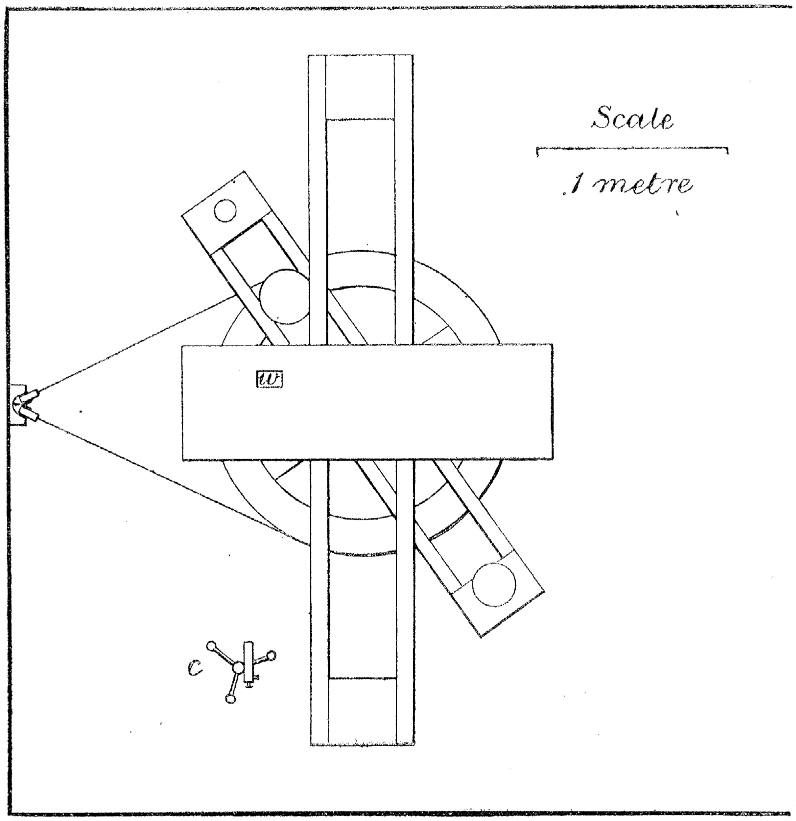
The temperature of the room was taken by means of a thermometer with a protected bulb at the end of a long wooden rod hanging down from the room above. The thermometer was about 6 feet from the floor, near one end of the case, and it could be rapidly pulled up into the room above and read by the observer before its temperature sensibly varied. The temperature never appeared to vary so much as $0 \cdot 1°$ C. in the course of two or three hours.

The Balance Case and its Supports.—The case (fig. 1) is a large cabinet of $1\frac{1}{4}$ inch wood, 1·94 metre high, 1·63 metre wide, ·61 metre deep, with three large doors in front giving access to the hanging masses and riders, and a small door at the back near the mirror hereafter described. It is lined inside and out with tinfoil, and under each of the suspended masses is a double bottom with a layer of wool between, making a total thickness of about $1\frac{1}{2}$ inch or 4 centims. At the top is a small window about 10 centims. square, through which the oscillations of the beam were observed. On each side within the case are placed three horizontal partitions, like shelves, to hinder circulation of the air.

The larger attracting mass and the attracted masses are gilded, and it is possible that some advantage may arise from having the surface of the case of different metal. For if it, too, were gilded, it would readily absorb radiation from the large mass, and when the inside temperature changed, the suspended masses would readily absorb radiation from the inner surface of the case. But gold probably absorbs considerably less of tin radiation than it absorbs of gold radiation, and so temperature changes are probably lengthened out more than if the case were gilded.

It was necessary to support the case so that the attracting masses could be moved about underneath it, and also to make it independent of the floor. Two brick pillars, 58 centims. × 36 centims. and 56 centims. high, were therefore built on thick beds of concrete under the floor, and about $3\frac{1}{2}$ metres apart. They rise up free from the bricked floor. Stretching between them are two parallel iron girders (g, g), about 30 centims. apart, and with their under side 56 centims. above the floor. The balance case is placed across the middle of these girders (see plan, fig. 2), with its

Fig. 2.



Plan of turn-table, girders, pillars, and balance case. *w*. Window in case. *c*. Usual position of cathetometer.

under surface level with that of the girders. The square base plate of the balance is placed on the girders on three levelling screws. Two horizontal screws attached to the girders bear against each edge of the base plate, so that it can be adjusted and fixed in any position.

To lessen vibration one tier of bricks is removed from each pillar, and in its place are inserted eight cylindrical blocks of indiarubber (i, i, fig. 1), originally 7·5 centims. diameter and 7·5 centims. high. These crushed down almost 1 centim. at once, but have not shown any further measurable contraction in the course of several years. Their effect in deadening vibration has been surprisingly great.

The Turn-table.—On a bed of concrete, and quite free from the brickwork of the floor, is a circular rail of cast iron, 1·3 metre in diameter. On this, on conical brass wheels and pivoted at the centre, runs the turn-table, about 1·5 metre in diameter. This is made of wood and covered with tinfoil. It is like a wheel with a flat circular rim, and with four flat spokes arranged as a cross. It is as nearly symmetrical as possible, and at opposite ends of a diameter are placed two shallow cups, in either of which the large attracting mass may rest. The centres of these cups are a distance apart, equal to the length of the balance beam. There are cut slots through the bottom of each cup, so that the bottom of the mass can be seen for the purpose of measuring the vertical diameter.

Two beams, 2·74 metres long, run across the turn-table 26 centims. apart, with the cups between them, and across the ends are two boards, each with a circular hole 12 centims. in diameter, and in either of these the smaller, or balancing mass, may rest. These beams are braced by brass rods to brass uprights at their middle points to diminish bending.

The turn-table is moved by an endless gut rope passing round it, and fixed at one point of the rim. The two sides of the rope pass over pulleys on to a drum in the room above. There are stops on the circular rail, against which come brass pieces on the turn-table when the masses are in position at either end of the motion. The drum can be turned easily by the observer at the telescope. Since the knife-edges and planes of the balance are of steel, all the moving parts of the apparatus are free from iron. As an illustration of the necessity of this, I may mention that for some time I used what I supposed to be a brass wire rope to move the turn-table, but on looking out for the explanation of some irregularities, I found that the brass was wrapped round a core of steel wire, which acquired poles at the highest and lowest points in the position in which it always rested between different sets of weighings. These poles had quite an appreciable action on the balance beam.

The Balance.—This is of the large bullion balance type, with gun-metal beam and steel knife-edges and plates. It was made specially for the experiment by Mr. Oertling, with extra rigidity of beam. Its performance has shown the great excellence of the design. The central knife-edge is supported on a steel plate by a frame-work rising 107 centims. above the base plate, and the usual moveable frame can be raised or lowered from outside the case, fixing the beam or setting it free to oscillate. The beam has often been left free to oscillate for months at a time, with the full load of 20 kilogrms. on each side, but I have no reason to suppose that the knife-edges have suffered at all.

The length of the beam was measured by taking the length of each half separately by a beam compass, and the mean of several measurements gave 123·329 centims. as the total length. The standard scale used throughout was that of a cathetometer made by the Cambridge Scientific Instrument Company. This scale has been verified at the Standards Office, and taking its coefficient of expansion as $\frac{1}{60000}$, it may be regarded for our purpose as perfectly correct at 18°, any errors being at that temperature much less than the errors of experiment. Comparing the beam compass with this scale, it was found that ·06 centim. must be subtracted, reducing the length to 123·269 centims. Now both beam and scale are of gun-metal and may, therefore, without serious error, be assumed to have the same coefficient of expansion, so that this is the length of the beam at 18°. At 0° it is 123·232 centims.

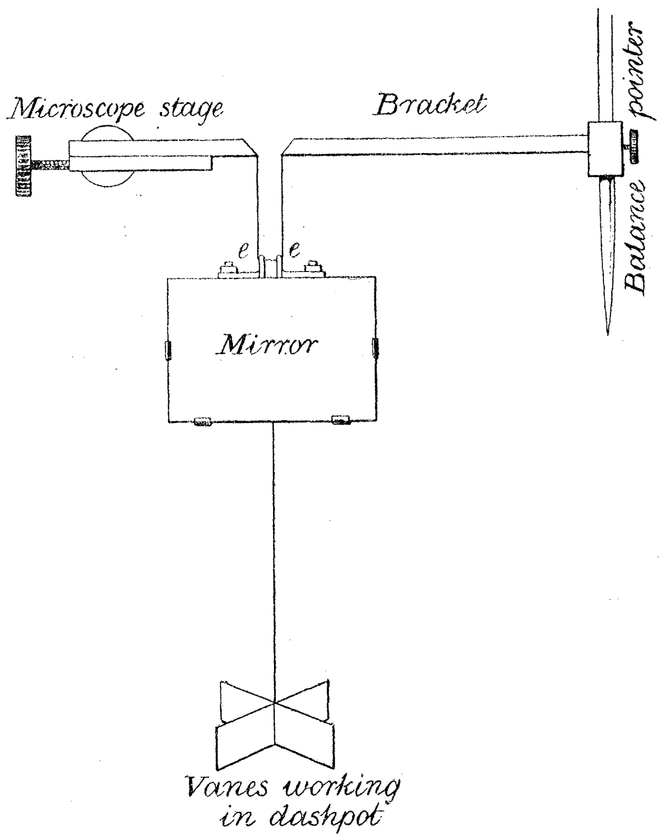
Mirrors, Telescope, and Scale.—At first a mirror was attached to the centre of the beam and the reflection of a scale in it was observed, either in the ordinary method or in the method described in the former paper ('Roy. Soc. Proc.,' No. 190, 1878), where a second fixed mirror is used to throw the ray of light a second, or even a third time back on to the moving mirror, each return increasing the deflection of the ray. But it was then necessary to make the time of vibration very long, and even when the time was three minutes, the tilt due to the attraction, *i.e.* the change of resting point, did not amount to more than two or three scale divisions. Now certain irregularities observed when the apparatus was first set up at Cambridge, led to experiments on the time taken by heat to get through the case in sufficient quantity to affect the balance, and I found that a coil of copper wire placed close under the case on one side (the bottom of the case being then solid, 1 inch thickness), heated by a current yielding 100 calories per minute, began to produce an appreciable disturbance on the balance in about 10 minutes, doubtless by the creation of air currents from the heated floor of the case. It appeared advisable, therefore, to reduce the time of a complete experiment to less than this if possible, and, consequently, the time of a single swing very much below 3 minutes. This could only be done if at the same time the optical sensibility were very greatly increased.

The employment of what may be termed the double-suspension mirror method due I believe to Sir WILLIAM THOMSON, and used by Messrs. G. H. and HORACE DARWIN in their experiments on the Lunar Disturbance of Gravity ('Brit. Assoc. Rep.,' 1881), has very satisfactorily solved the problem, giving a greatly increased deflection on the scale, even when the time of oscillation is as short as twenty seconds.

This method, which deserves to be more generally known and applied for the detection of small motions, consists in suspending a mirror by two threads, one from a fixed point, the other from the point which moves. The angle through which the mirror turns for a given motion of the latter point is inversely as the distance between it and the fixed point, so that by diminishing this distance the sensibility of the arrangement may be almost indefinitely increased.

To apply it to the balance, a small bracket (fig. 3) is fixed to the ordinary pointer of the balance, about 60 centims. below the central knife-edge. This projects horizontally at right angles to the axis of the beam, and it is bevelled at the edge. Close to it is another bevelled edge attached to a microscope stage movement which is fixed on to the central pillar of the balance. A thread of silk (as supplied for the Kew magnetometer) is fastened to the stage, passes over the bevelled edge, through two eyes, *e e*, on a light frame holding the mirror up over the bevelled edge of the bracket, and is fastened to the bracket. The microscope stage movement allows the distance between the threads to be adjusted, and also enables the azimuth of the mirror to be altered.

Fig. 3.



Double Suspension Mirror (half size).

Of course, if the mirror were weightless, it would not affect the sensibility of the balance, and the threads might be brought very close together. But the weight of the mirror—it is silver on glass, 56 millims. × 38 millims. × 10 millims.—has a considerable effect on the sensibility, diminishing it with decrease of distance between the points of suspension. In practice it has been found convenient to work with the threads parallel, and from 3 to 4 millims. apart, the time of swing one way being adjusted to about 20 seconds. A less time hardly suffices for a correct determination and record of the scale reading. Taking 4 millims. as the distance, and supposing the bracket to be 600 millims. below the knife-edge of the balance, the mirror evidently turns through an angle 150 times as great as that through which the beam turns.

The drawback to this method of magnification is that the mirror has its own time of swing and is easily disturbed. The swings of the mirror and the disturbances are, however, effectually damped by having four light copper vanes attached to the end of a thin wire, projecting down from the mirror and working in a dash pot with four radial partitions not quite meeting in the centre, one vane being in each compartment. I found that mineral lubricating oil is very suitable for the dash pot, as the surface keeps quite clean and there is little evaporation. The swings of the balance are also very greatly damped by this arrangement, but the effect of this will be discussed later.

The telescope and scale are in the room over the balance room (see fig. 1), a hole being cut through the floor, and a small glass window being fixed in the top of the case. As the suspended mirror is in a vertical plane it is necessary to have an inclined mirror fixed in front of it to direct the light from the scale horizontally on to it and back again to the telescope. With the magnification used it was necessary for good definition to have an exceedingly good inclined mirror, and several were rejected before a suitable one was obtained. That finally used is a silver on glass oval mirror, 60 millims. × 40 millims., by BROWNING. The glass window in the case is optically worked and carefully adjusted to be normal to the path of the light.

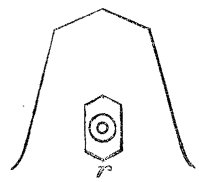
The telescope has a 3-inch object glass of about 4 feet focal length. It is fixed on a brick pillar built on one of the brick arches, which form the ceiling of the balance room, and it rises free from the floor of the observing room. To destroy vibration one course of bricks is replaced by blocks of india-rubber. The scale has 50 divisions to the inch (say $\frac{1}{2}$ millim.), ruled diagonally, and divided to tenths by cross lines. It is photographed on glass from a scale drawn on paper with very great care, 50 inches long (say 127 centims.), and with 500 divisions. The photograph is $\frac{1}{10}$th of this length, and only the central part of the scale, about 60 divisions in length, has been used. The diagonal ruling enables a tenth of a division to be read with certainty, and the readings recorded in the Tables, pp. 625–655, are in tenths. Though the lines appear somewhat coarse, I have not been able to find another scale equal to it in distinctness and in ease of reading. As all the results depend on the ratio of measurements, taken almost simultaneously, of deflection due to attraction and rider respectively, in the same part of the scale, I have not thought it necessary to calibrate it.

The scale is fixed horizontally on the end of the telescope close to the object glass with a piece of ground glass over it. It was illuminated in general by an incandescent lamp placed above it, once by an Argand burner.

The distance from the scale to the mirror and back is about 5 metres. It follows that 1 division of the scale corresponds to an angular motion of the mirror through ·0001 radian. But this is at least 150 times the angle through which the beam turns for the same deflection. So that 1 scale division implies an angular motion of ·0000006 radian, or $\frac{2}{15}''$ in the beam. As the total length of swing in Table III. is never more than 12 divisions, the angular vibrations of the beam are at the most about $1''\cdot6$, and the linear vibrations of the masses, since the half beam is about 60 centims., are at the most about ·005 millim. This shows that it is quite unnecessary to consider any change of distance due to vibration. The greatest deviation from the mean in any of the series of weighings recorded is about 1 per cent. of the rider value, corresponding to about $\frac{1}{10}$th of a division, or an angle of $\frac{1}{75}''$ in the beam, and a distance of ·00004 millim., say $\frac{1}{600000}$ inch, in the motion of the masses. This seems to show that the method is accurate as well as sensitive.

Determination of the Value of the Scale Divisions by means of Riders.—This was done by means of centigramme riders (fig. 4), these being the least weights which

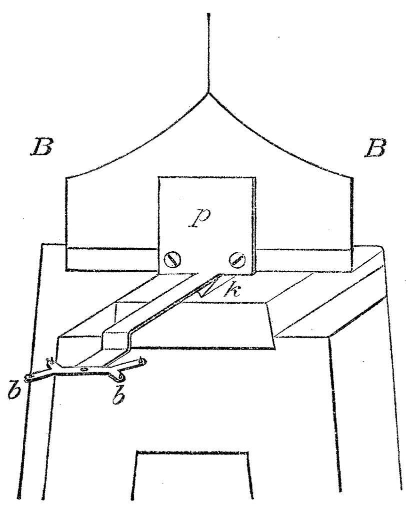
Fig. 4.



Rider, actual size, and end of lifting rod, *r*.

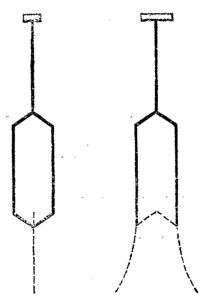
appeared capable of sufficiently accurate determination. Instead of transferring the same rider from point to point, it was much easier to use two equal riders, and to take one up while the other was being let down a given distance from it. The distance selected was about 2·5 centims., since the deflection due to the transfer of one centigramme so far along the beam was nearly equal to that due to the greatest attraction to be measured.

Fig. 5*a*.



Subsidiary rider beam, *bb*, attached to centre of balance beam, *BB*, by plate *p* just above central knife-edge, *k* (half size).

Fig. 5*b*.

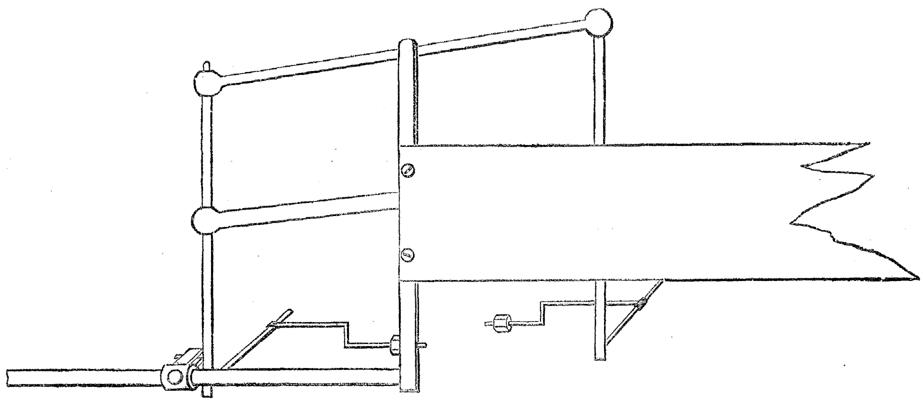


Wire frames depending like scale pans from ends of *bb*, fig. 5*a*, side and end views (half size).

At first the riders when on the beam rested in V notches in a pair of parallel brass strips fixed on and parallel to the beam. But this plan was soon abandoned, as there was no certainty about the position of the rider in the notches. The riders were then supported in little wire frames, each hung by two cocoon fibres from the edges of a plate fixed to the beam, the edges being parallel to the central knife-edge. The only objection to this method was the very considerable time spent in replacing the fibres after the breakages which occurred on dusting or any readjustment of the balance.

Ultimately a small subsidiary beam, about 2·5 centims. long, was attached to the centre of the balance beam just above the knife-edge (fig. 5*a*), the scale pans being represented by small wire frames in which the riders could rest (fig. 5*b*). These frames depend from agate pieces resting on steel points at the extremities of the subsidiary beam in the way now usually adopted in delicate assay balances. This mode of supporting the riders appears to be perfectly satisfactory.

Fig. 6.



Lifting rods to raise or lower riders (half size).

To raise or lower the riders two short horizontal lifting rods parallel to the beam move up and down within the supporting wire frames with a nearly parallel motion, and on them are two metal pieces with their upper surfaces shaped so that the riders rest on them without swinging (fig. 4, *r*). They are the extremities of L-shaped projections from a jointed parallelogram framework (fig. 6), supported on an upright in front of the subsidiary beam. The framework is moved by a tongue engaging with it, and projecting from a horizontal rod, which rotates about its axis in bearings, one within the case and the other outside. The rod is turned through an angle of about 30° between stops by an endless string passing upwards and round a wheel in the observing room.

The parallelogram framework and the bearing of the rotating rod within the case are both supported independently of the case from the ceiling. At first they were supported respectively on the central pillar of the balance and on the case; but when the increase of optical sensitiveness enabled me to detect small irregularities, I realised how essential it was for accurate weighing that all parts of the apparatus moved from the outside should be supported quite independently of the balance. Even the string moving the rod transmitted great and continual vibration. The rod and the framework with the lifting levers were, therefore, supported by iron rods coming down from the ceiling through holes in the top of the case, large pieces of cardboard stretching from these rods over the holes to hinder the passage of dust into the case. Once or twice in the course of preliminary experiments irregularities were traced to accidental contact of outside bodies with the case.

It appeared just possible that there might be electrification of the riders by friction with the lifting rods, especially when they were supported by cocoon silk. It was, therefore, advisable that the surface of the lifting rods should be of the same kind as that of the riders. As the latter are silver wire gilded, the lifting rods are also gilded. It may not be uninteresting to note here a curious phenomenon which occurred during some early preliminary experiments. The shaped pieces on the lifting rods were then of wood covered with gold leaf, put on with ordinary paste. After they had been on for some months, I obtained some very various results for the deflection due to the riders, and on examining the lifting rods I found that a number of long needle growths projected from the wood pieces and interfered with the supporting wire frames. At first I thought these were organic, but my colleague, Professor HILLHOUSE, examined them and found that they were crystalline. Doubtless, the hygroscopic paste set up electric action between the gold leaf and the brass to which the wood pieces were attached, and the crystals were probably zinc sulphate. The wood was then replaced by brass gilded, and no further difficulty of the kind was experienced.

The length of the subsidiary beam was kindly determined for me by Mr. GLAZEBROOK at the Cavendish Laboratory. The steel points are hardly sharp enough to determine the distance to 1 in 10,000, but the mean of the results is sufficiently exact. The following are Mr. GLAZEBROOK's determinations; the four points being denoted by *abcd* :—

Date.	Temperature.	Number of readings.	*a* to *b*.	Number of readings.	*c* to *d*.
	°		inches.		inches.
1889 July 4 .	22·5	6	·9985	6	·9979
July 11 .	21·5	3	·9990	3	·9982
July 12 .	23	3	·9988	3	·9979

These are in terms of a gun-metal standard of which the error is only 3 in 100,000 at 0°, and, therefore, for my purpose negligible. The beam is of brass, and we may assume with sufficient exactness that it has the same expansion as the standard. The temperature may, therefore, be left out of account. The mean value of $\frac{1}{2}(ab + cd)$ is therefore ·9983375 inch, or taking 2·539977 centims. to the inch we obtain

Length of beam at 0°, 2·53575 centims.

There is an advantage in fixing this beam at the centre, which should be noted here. Suppose the riders are not quite equal, but have values w and $w + \delta$. Let the two ends of the subsidiary beam be distant a and $a + l$ from the central knife-edge. Then the effect of picking up the rider w from the nearer, and letting down the rider $w + \delta$ on the further end, is equivalent to putting at unit distance

$$(w + \delta)(a + l) - wa = wl + \delta(a + l) = wl\left(1 + \frac{\delta}{w}\frac{a + l}{l}\right),$$

or the error δ/w is multiplied by $(a + l)/l$, and, if the beam is not central, $(a + l)/l$ may be greater than 1, so that the error is magnified.

If, however, the small beam is central, l is equal to $-2a$, and the error is multiplied by $+\frac{1}{2}$.

If the riders are interchanged and the weighings are then repeated, the mean result is the same as if riders with the mean value were used for

$$w(a + l) - (w + \delta)a = wl - \delta a = wl\left(1 - \frac{\delta}{w}\frac{a}{l}\right),$$

and the mean of this and the above is

$$\left(w + \frac{\delta}{2}\right)l.$$

The Attracting and Attracted Masses.—These are all made of an alloy of lead and antimony, for the sake of hardness, the specific gravity in each case being about 10·4. They were made at various times and places, the large attracting mass M being made more than 12 years ago by Messrs. STOREY, of Manchester. The smaller balancing mass m was made in 1889 by Messrs. HEENAN, of Manchester and Birmingham. These were both cast with a "head" on, and then turned. The attracted masses A and B were made by Messrs. WHITWORTH, and subjected to hydraulic pressure before turning. The dimensions have been measured from time to time, and there is no evidence of any sensible change of shape.

The larger mass M and the attracted masses A and B were weighed at the Mint through the kindness of the Deputy Master and Professor ROBERTS-AUSTEN. For the weight of the balancing mass m, I am indebted to Messrs. AVERY, of Birmingham. The large mass M has suffered two accidents since it was weighed, once being slightly cut into by a saw during some alteration of the case, and once being scratched by coming into contact with a piece of metal fixed to the turntable in taking it out of its place. The saw-cut was carefully filled in with lead, and the scratch removed only a fraction of a gramme, as was determined by taking a mould of the hollow. I should be glad to think that the determination of the attraction was sufficiently exact to make reweighing necessary, but I am afraid that the alteration in weight is very far beyond the important figures, and I therefore take the original weight as sufficiently near the truth. The masses A and B have been gilded since the original weighing, but I carefully determined their increase of weight by the balance used in the gravitation experiment.

The values given below in the second column are the true masses. In the third column are the masses of M and m, less the air displaced by them, this being taken as 18·41 and 9·2 grms. respectively. It will be shown later that the true masses of A and B and the reduced masses of M and m may be used in the calculations of the result.

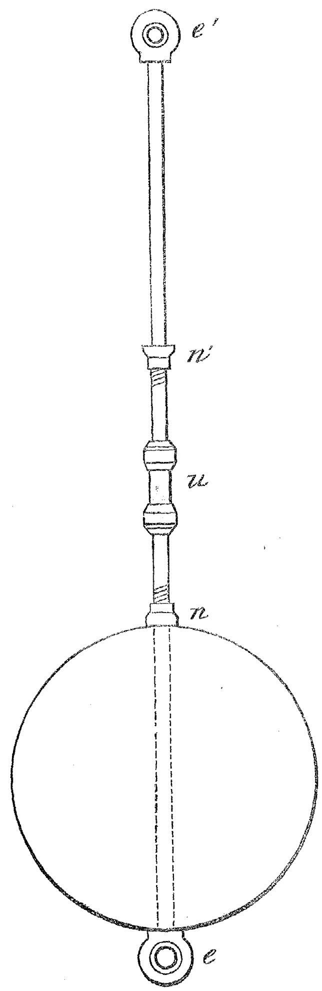
	True mass in grammes.	Mass less that of air displaced.
M	153407·26	153388·85
m	76497·4	76488·2
A	21582·33	
B	21566·21	

Suspension of the Attracted Masses.—Each of the attracted masses is drilled through along a diameter, the hole being ·6215 centims. in diameter, and a brass rod (fig. 7) terminating in an eye e below, is passed through the hole. The mass is secured in position by a nut n working in a screw thread cut for a short distance in the rod. An exactly similar rod terminating in a similar eye e', and with a similar nut n', is fastened end to end to this by a union u. The nuts and the inner sides of the enlargements for the eyes are hollowed out so as to fit exactly on to the spheres.

From the ends of the balance beam hang down stout brass wires terminating in hooks. If these hooks are passed through the eyes e' the attracted masses are close to the floor of the balance case, and their centres are adjusted to be about 32 centims. from the centre of the large attracting mass when under either of them. If the masses are turned over so that the hooks pass through the eyes e, they are about 30 centims. higher or at nearly double the distance, the length ee' being about 48 centims. The rods being perfectly symmetrical about the union u, the attraction on them is the same in either position. The weight of each is about 212 grms., or about $\frac{1}{100}$ of the attracted mass, so that any small variation in their position would produce a negligible variation in the total attraction. By the differential method, the attraction on them entirely disappears from the results.

The mode of support of the attracting masses M *and* m.—This has already been described when describing the turntable.

Fig. 7.



Suspender for Attracted Mass (one-fourth size).

The Riders.—Four centigrm. riders, A, B, C, D, of silver wire gilt were made by Mr. OERTLING of the form shown in fig. 4. These were weighed in 1886 at the Bureau International des Poids et Mesures, by M. THIESEN. The following is an extract from the certificate :—

"*Densité et volume.*—Comme densité on a accepté celle de l'argent, et par conséquent comme volume de chacun des cavaliers, 0·0010 millilitre.

"*Détermination des poids des cavaliers.*—L'étude des poids de ces quatre cavaliers a été faite par M. le Dr. THIESEN, adjoint du Bureau International, chargé de la section des pesées. M. THIESEN au moyen de la balance STÜCHRATH, destinée à des poids au dessous du gramme, a d'abord déterminé les différences entre les quatre cavaliers pris deux à deux dans les six combinaisons possibles, et ensuite la différence entre l'ensemble des quatre cavaliers et le poids de 40 milligrms. de la série 0 du Bureau, série en platine iridié récemment étalonnée par M. THIESEN. Les comparaisons ont été faites du 19 au 29 Mars, 1886

"*Résultats.*—De l'ensemble de ces comparaisons résultent les poids :—

$$\begin{aligned} A &= 10{\cdot}1247 \text{ milligrms.} \\ B &= 10{\cdot}0615 \quad ,, \\ C &= 10{\cdot}1196 \quad ,, \\ D &= 10{\cdot}1262 \quad ,, \end{aligned}$$

"L'incertitude de ces déterminations ne dépasse pas 0·001 milligrm."

A and D were selected for use as being the nearest to each other in value. B and C were kept untouched in boxes till 1890. In the various experiments made between 1886 and the final weighings, A and D had necessarily been handled to some extent, especially through the frequent breaking of the silk fibre suspension used before the subsidiary beam described above, and it appeared possible that their weights might be altered. It was also necessary to determine whether an appreciable amount of dust was deposited on them in the course of several weeks as it was inconvenient to dust them frequently. The riders B and C might be assumed to have the same weight as in 1886, and could be taken as standards.

To make the weighings a 16-inch chemical balance was arranged with a double suspension mirror on exactly the principle already described for the large balance. The apparatus was put together quickly with materials at hand, and might easily be greatly improved. It is only described here to show how accurate the method is, even with such rough apparatus, and that it is applicable to a small as well as a large balance.

A cork sliding on the pointer with a horizontal needle stuck in it, served to support one thread of the mirror; a stand with a projecting arm—one made to hold platinum wires in a Bunsen flame—served to support the other thread. A wire with a small copper vane depended from the mirror and was immersed in an oil dashpot. The telescope and a millimetre scale were on a level with the mirror about 2 metres distant on one side of the balance. Two brass strips, parallel to each other and the beam, were fixed on the top of one arm of the beam, and in each of these were two V notches in which centigramme riders could rest. Two levers, worked by cams on a rod rotated by the observer, picked one rider up and let down the other, so that the effect was equivalent to the transfer of 1 centigrm. from one notch to the other. Their distance apart was such that this was equivalent to the addition of ·3284 milligrm. to one pan of the balance. This was the arrangement described in my former paper. Attached to one pan was a pair of brass strips parallel to each other, and such that the riders A, B, C, or D, would just rest across them. Two lifting rods worked up and down between these strips, so that of the two riders to be compared, one could be picked up at the instant the other was let down. The lifting rods were worked by a rod rotated by the observer and supported quite independently of the balance, and of the slab on which it rested. By this plan the value of the scale divisions and the shifting of the centre of swing on changing the weights to be compared, could all be determined without raising the beam of the balance between the successive weighings, an essential condition, I believe, for exact work.

The weighings were made in the large room of the Physical Laboratory, and no precaution was taken to protect the balance case beyond placing a board in front of it. The room is draughty and subject to great variations of temperature, so that the weighings were made under very disadvantageous circumstances. One result of this was a rapid and sometimes very great change of resting point in the course of a few hours, so that the scale passed out of the field of view. In order to bring it back without opening the case, two glass tubes passed through the top of the case, almost down to the scale pans, and small bits of wire could be dropped through these on to either pan as needed. Caps fitted on to the tubes to prevent draughts. This plan appears worthy of mention, as it suggests a mode of determining the value of a scale division when a balance is either too sensitive for riders or has no special arrangement for their accurate use. If a piece of wire weighing, say, 1 milligrm. is cut into say ten nearly equal parts, and if these are dropped on to the two pans alternately the shiftings of the centre of swing will be to and fro, about equal distances, due to about ·1 milligrm., but the sum of the shiftings will be that due to 1 milligrm., and the balance at the end will be nearly in the same position as at the beginning.

The following is an abstract of the comparisons of the riders. They were made soon after the first determinations of attraction on February 4, when A and D had not been dusted for three months.

In each case three extremities of swing were observed, and the centre of swing was determined from these by the graphic construction described later (p. 595).

The centres of swing were combined in consecutive threes in the usual way to give the differences in scale divisions.

Thus, in the first series, the successive centres of swing with D and A alternately in the scale pan were

D	A	D	A	D
231	223	217	211·9	208

whence

$$(D - A)_1 = \frac{217 + 231}{2} - 223 = + 1·0 \text{ division.}$$

$$(D - A)_2 = 217 - \frac{223 + 211·9}{2} = - 0·45 \text{ division.}$$

$$(D - A)_3 = \frac{217 + 208}{2} - 211·9 = + 0·6 \text{ division.}$$

$$\text{Mean } D - A = ·38 \text{ division.}$$

Successive values of the differences alone are given below.
The time of swing one way was about 16 seconds.

February 16, 1890.

(1.) COMPARISON of A and D, undusted.

Deflection due to ·328 *milligrm.* 83·45, 82·45, 84·45 divisions. Mean 83·45 divisions.

$$D - A = 1{\cdot}0, \ - {\cdot}45, \ + {\cdot}06 \text{ division. Mean } {\cdot}38 \text{ division};$$

therefore

$$D = A + {\cdot}0015 \text{ milligrm.}$$

(2.) COMPARISON of A undusted, D dusted.

Value of scale division taken as in the last.

$$D - A = - {\cdot}5, \ + {\cdot}3, \ - {\cdot}1, \ - {\cdot}4, \ + {\cdot}25. \text{ Mean } - {\cdot}09 \text{ division};$$

therefore

$$D = A - {\cdot}0004 \text{ milligrm.}$$

February 17, 1890.

(3.) COMPARISON of A and D, both dusted.

Value of scale division taken as below (4).

$$D - A = - {\cdot}1, \ - {\cdot}2, \ + {\cdot}3, \ - {\cdot}3, \ - {\cdot}8. \text{ Mean } - {\cdot}22 \text{ division};$$

therefore

$$D = A - {\cdot}0008 \text{ milligrm.}$$

(4.) COMPARISON of C and D.

Deflection due to ·328 *milligrm.*, 85·35, 85·4, 84·65. Mean 85·13 divisions.

$$D - C = + {\cdot}15, {\cdot}00, \ + {\cdot}05, \ - {\cdot}15, \ + {\cdot}05, \ + {\cdot}3, \ + {\cdot}05, \ - {\cdot}05, \ - {\cdot}35, {\cdot}05, {\cdot}35, \ 45, {\cdot}50, {\cdot}2. \text{ Mean } {\cdot}114 \text{ division};$$

therefore

$$D = C + {\cdot}00044 \text{ milligrm.}$$

February 18, 1890.

(5.) Comparison of C and D repeated.

Deflection due to ·328 *milligrm.*, 92·75, 92·3, 91·65. Mean 92·23 divisions.

D − C = ·35, − ·05, − ·8, − ·95, − ·1, + ·05, 0, − ·15, − ·1, + ·05. Mean − ·17 division;

therefore

$$D = C - \cdot 0006 \text{ milligrm.}$$

Combining this with the last, and weighting them in the ratio of the numbers of determinations in each,

$$D = C + (\cdot 00044 \times 14 - \cdot 0006 \times 10) \div 24 = C - \cdot 0000 \text{ milligrm.}$$

(6.) Comparison of A and D.

Value of scale division taken as above, ·328 milligrm. = 92·23 divisions.

D − A = ·45, ·25, ·1, − ·2, − ·1, ·35, ·25, ·45, ·60, ·5, ·5, ·55, ·5, ·75, ·55, ·1, ·05, ·45, ·10, ·30, ·8, ·9, ·35, ·2, ·30, ·55, ·50, ·35, ·45, ·45. Mean ·378 division;

therefore

$$D = A + \cdot 00134 \text{ milligrm.}$$

Examining the values obtained in (1), (2), and (3), it will be seen that no trustworthy evidence is given of a difference due to dusting. Any existing difference was probably under ·002 milligrm, and since the weighings on February 4, before dusting, were made with the attracted masses in the upper position, when the attraction was only one-fourth of that on which the final results depend, we may safely neglect the effect. After this the riders were dusted more frequently, so that we may probably assume their values more constant.

The comparisons of C and D, and of A and D, in (4), (5), and (6), were made more carefully. That of A and D in (6) is much the best of the series, the air in the laboratory happening to be steadier while it was made. The range between the greatest and least values of the difference is one scale division, or ·0036 milligrm., and the different results are grouped fairly closely about the mean.

The centres of swing and the differences are plotted in Diagram VIII. I do not claim that these results show any remarkable accuracy when compared with those obtained at the Bureau International des Poids et Mesures, but remembering how rough the apparatus was, and how little precaution was taken to ward off air currents, I have not the slightest doubt that, with special design of apparatus and more suitable locality, the results could be very greatly improved, and the accuracy carried far beyond anything hitherto reached. As they stand, they seem to show the value of the combination of a short time of swing with optical magnification.

The result of comparisons (4), (5), and (6), is, that if C has its Paris value, viz., C = 10·1196 milligrms., then, A = 10·1183 milligrms., and D = 10·1196 milligrms. ; whence $\frac{1}{2}(A + D) = 10{\cdot}119$ milligrms. This value may be used in calculating the result, since the riders were interchanged before Set II. was taken.

The losses experienced since 1886 by A and D are respectively, by A ·0064 milligrm., and by D ·0066 milligrm., *i.e.*, they have diminished by practically equal amounts. This was to be expected as they have probably received equal amounts of rough usage.

The substitution of the subsidiary beam for the cocoon fibre suspension of the riders having greatly diminished the handling to which they were subjected, I have not thought it necessary to weigh them again during the work.

Linear Measurements.

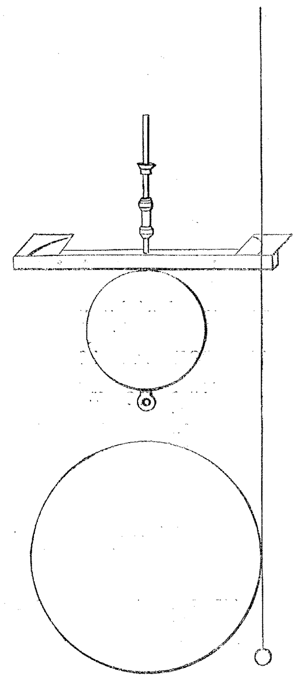
In the mathematical theory it will be shown that the lengths required are those marked in fig. 15, viz., the horizontal distances, L and l, and the vertical distances, D_1 D_2, d_1 d_2, H_1 H_2, h_1 h_2.

The Horizontal Distances.—Except when estimating the moment of the rider, the distance L is really that between the verticals through the centre of M and the centre of the more distant attracted mass. But the verticals through the centre of M in each position, so nearly passed through the centre of the mass above it, and, therefore, through the knife-edge from which it hung, that L was taken as equal to the length of the beam (p. 571).

The accuracy of this adjustment was secured as follows. A horizontal cross-piece was fixed on the top of each attracted mass, with two horizontal cards at its two ends, each with a portion of a circular arc on it, with radius equal to that of the large mass M, and with centre over that of the attracted mass (fig. 8). A plumb line was then hung just in front of the case, and the balance was moved by the horizontal screws bearing against the base plate until the plumb line always appeared to touch the circular arc above, when it appeared to touch the large mass below. The adjustment was not quite perfect, but the error in the worst case was probably not more than 1 millim., and certainly less than 2 millims. Such an error in the horizontal distance is negligible.

The distance l had different values for the two positions occupied by m on the turntable. Calling these values l_1 and l_2 respectively, $l_1 + l_2$ was found by measuring a, the inside distance between M and m, arranged as in Set II., and b, the inside distance between them, when m was put on the same side of the turntable as M, and adding to $a + b$ the sum of the diameters of M and m in the radial direction of the turntable as taken by square calipers.

Fig. 8.



Plumb line Adjustment of Masses.

The following are the values in terms of the cathetometer scale already referred to, the temperature being 15° C.:—

$$
\begin{aligned}
a &= 157{\cdot}01 \\
b &= 33{\cdot}95 \\
\text{Diameter of M} &= 30{\cdot}52 \\
\text{,, \quad ,, \quad } m &= 24{\cdot}23 \\
\hline
\text{therefore} \qquad l_1 + l_2 &= 245{\cdot}71
\end{aligned}
$$

The value of $l_1 - l_2$ was found by measuring the shortest distance of m from the wall when respectively in the first and the second positions on the turntable. It was found that

$$l_1 - l_2 = \cdot 12$$

whence

$$l_1 = 122{\cdot}915$$

$$l_2 = 122{\cdot}795.$$

We may obtain from these measures an independent value of the radius of the circle in which the centre of M moves. With perfect adjustment this should be $\frac{1}{2}\mathrm{L} = 61{\cdot}66$ at 18°.

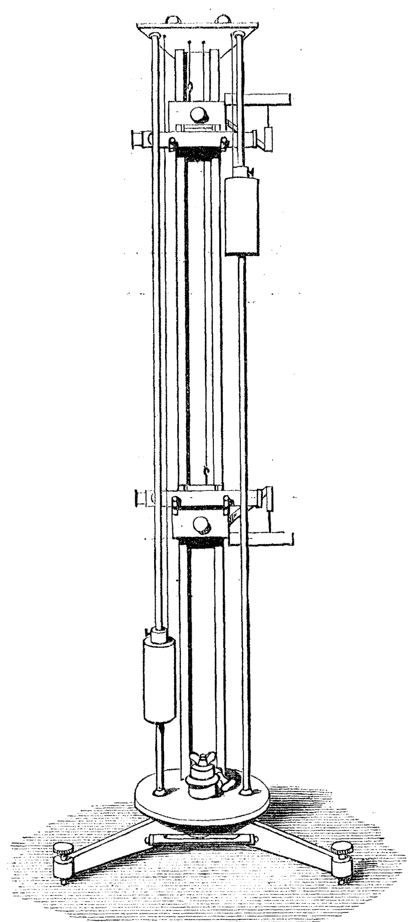
It is equal to a + radius of M + radius of $m - l_2$, or, by the above measures,

$$= 157{\cdot}01 + 15{\cdot}26 + 12{\cdot}115 - 122{\cdot}795$$
$$= 61{\cdot}59,$$

which is only ·07 centim. less than $\frac{1}{2}$ L.

Inasmuch as the wood probably expanded less than the cathetometer scale, while the metal expanded more, I have assumed as a rough approximation that the total expansion equalled that of the scale, so that the values of l_1 and l_2 are correct at 18° (see p. 571). No importance is, however, to be attached to this temperature correction.

Fig. 9.



Cathetometer used to measure Vertical Diameters.

The Vertical Distances.—At the conclusion of each set of weighings with the attracted masses in a given position, the vertical distances between the top of the attracting masses and the bottom or top of the attracted masses (accordingly as they were in the upper or lower position) were measured by the cathetometer already referred to.

This instrument is of the well-known design of the Cambridge Scientific Instrument Company, and is especially adapted for measuring differences of level at different distances in different vertical planes. It reads to ·002 centim. The account of these measurements will be found in Table II. (p. 614, *et seq.*).

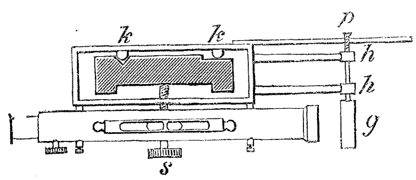
To find the distances D, d, H, h (fig. 15), it was necessary to add to the actual distances measured the sum or difference of the vertical radii of the attracting and attracted masses, and, therefore, the vertical diameters of all the masses were measured.

For this purpose I used a cathetometer which has lately been constructed for me by Messrs. BAILEY, of Bennett's Hill, Birmingham. I have to thank Mr. POTTS, of that firm, for his care in its construction, and also for the trouble which he has taken in the construction and alteration of much of the apparatus used throughout the work recorded in this paper. As the cathetometer is, I believe, new in design and satisfactory in its performance, it appears worthy of description.

The Cathetometer used to measure Vertical Diameters (fig. 9).—There are two telescopes, one to sight the upper the other to sight the lower of the points between which the vertical height is required. There is no scale on the instrument, but after the telescopes are fixed to sight the two points the instrument is turned round a vertical axis, so that the telescopes sight a vertical scale at the same distance from them as the points. In general, of course, the cross wire will appear to lie between two divisions, but by means of the fine adjustment, to be described below, the two nearest scale divisions are brought in succession on to the cross wire, and by interpolation the reading corresponding to the point first sighted by the telescope is determined.

The telescopes are fixed on collars running up and down the main pillar, which has a section of the form shown in fig. 10 (shaded).

Fig. 10.

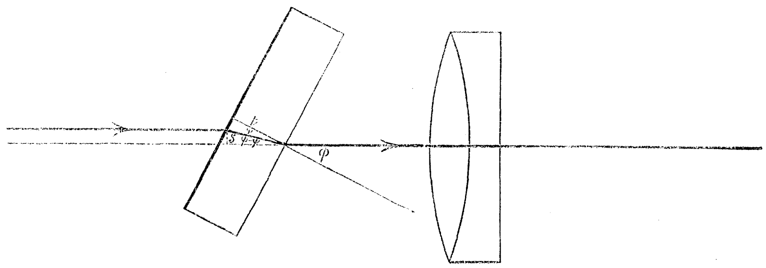


Section of pillar and collar of new Cathetometer.
s, clamping screw. k, k, guiding knobs. g, glass plate for fine adjustment, turning on axis h, h, with pointer at p perpendicular to plane of figure.

The guides consist of three knobs, k k, on the inside of the collar, two sliding in a vertical V-groove and one on a plane, both groove and plane being at the back of the pillar. A screw, s, clamps the collar in any position. Gut strings running up over pulleys and supporting counterpoises, sliding on the thinner pillars (see fig. 9), are attached to the collars so that these move easily. At first springs were used to keep the knobs always in contact, but I found it much better to remove these and trust merely to hand pressure to keep the collars in the proper position before clamping with the screw s.

The fine adjustment is secured by the use of a piece of plate-glass placed in the front of each object glass (g, fig. 10), and capable of rotation about a horizontal axis, $h\ h$. A pointer is fixed on the end of this axis at p, and at its end is a small glass plate with a scratch on it moving close against a straight scale. If the plate is initially normal to the optic axis of the telescope, on turning it through ϕ, the ray which now comes along the optic axis has been shifted by transmission through the plate parallel to itself, a distance $t \sin(\phi - \psi)/\cos\psi$, where t is the thickness of the plate and ψ is the angle of refraction within it (see fig. 11).

Fig. 11.



Section of fine adjustment plate.

This shifting happens for small angles to be nearly proportional to $\tan\phi$, and, therefore, to the reading on the straight scale. To show how nearly this is the case the following table gives the shifting for angles of 5°, 10°, and 20°, with a thickness of $t = 1$ centim. and a refractive index $\mu = \frac{3}{2}$:—

Angle $\phi =$	Shifting.
5°	$\frac{1}{3}\tan 5°\ (1 - \cdot 00042)$
10	$\frac{1}{3}\tan 10°\ (1 - \cdot 00156)$
20	$\frac{1}{3}\tan 20°\ (1 - \cdot 0052)$

The error in taking the shifting as proportional to $\tan\phi$ is, up to 20°, quite negligible in ordinary telescope-cathetometer work. If it is desirable to have greater accuracy, it is probably best to use a table of corrections to the tangent; but it is possible to get an exact scale thus :—

Let OP, fig. 12, represent the pointer of length r, making ϕ with a line MN. Let a pointer PM jointed to this at P be of length μr, and let its extremity M move on the line MN. Drawing OD at right angles to MN, if s is the shifting, we have

$$\mathrm{OD} = \mathrm{OP}\,\frac{\sin(\phi - \psi)}{\cos\psi}$$

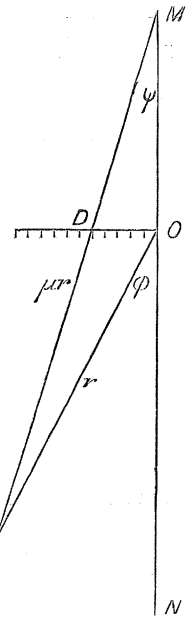
$$= \frac{rs}{t},$$

or

$$s = \frac{t}{r}\ \mathrm{OD}.$$

Probably the practical difficulties in the use of such an arrangement would render it troublesome and uncertain.

Fig. 12.



The plate is used as follows:—Adjust it normal to the optic axis of the telescope, and move the telescope till the required point is brought as near to the cross-wire as is possible by the hand. Clamp the telescope, and then turn the plate till the point is exactly on the cross-wire. Read the position of the pointer attached to the plate on its scale. Repeat these operations with the other telescope on the other point, then turn the instrument about its vertical axis till the telescopes sight the vertical scale placed at the same distance away as the two points. Looking through one of the telescopes the cross-wire is in general not exactly on a division. Turn the plate so that first the nearest division above, and next the nearest division below, is on the cross-wire. Reading the position of the pointer in each case, interpolation gives us the reading on the vertical scale corresponding to the position of the pointer when the cross-wire was between the two scale divisions. Doing this for each telescope the difference between the two points is found in terms of the vertical scale.

The plates I have used are about 9 millims. thick, and the pointers about 9 centims. long. They move over scales such that 25 to 27 divisions correspond to a shifting of 1 millim. The lower scale is graduated from 0 to 50, the upper from 50 to 100 to prevent confusion. The 50 divisions occupy a distance of 66 millims.

It will be observed that in this form of instrument the level error is practically entirely obviated. It can only come in if the scale is not at the same distance as the height to be measured, and may then be made negligible in practice by levelling the telescopes. Indeed, the uncertainty of measurement appears only to depend on the uncertainty with which the cross-wire can be brought to the proper point, that is, it depends only on the magnifying power and definition of the telescopes used.

To illustrate the use of the instrument, a full account of the determinations of the vertical diameters is given in Table II. Below are the results, and for the sake of showing that there has certainly been no great change in shape, I give results obtained with a cathetometer more than 10 years earlier at the Cavendish Laboratory at Cambridge.

	1890.	1880.
	centims.	centims.
Large attracting mass M . .	30·526	30·5192
Small „ „ m . .	24·176	
Attracted mass A	15·8203	15·8166
„ „ B	15·7829	15·7842

The diameters of M and m in a horizontal direction parallel to a radius of the turntable measured by square callipers were

$$M = 30{\cdot}52 \text{ centims.}$$

$$m = 24{\cdot}23 \quad \text{,,}$$

Temperature Correction.—Though the expansion of the masses was to be expected of an unimportant amount, I thought it advisable to attempt to measure it, in case there might be anything anomalous. One of the attracted spheres, B, was for this purpose placed between two vertical levers, in a tank through which could be run a continuous stream of cold or warm water. These levers depended from horizontal rods which could rock or slightly rotate on fine point suspensions. This was, in fact, a kind of double Lavoisier and Laplace apparatus. The motion of each lever was shown by another lever of about the same length, rising vertically up from each horizontal axis, and serving as the moving support for a double suspension mirror in which was viewed the reflection of a millimetre scale. Two telescopes and one scale were used for the two mirrors, though it would not have been difficult to arrange one telescope and two scales. The value of one scale division was determined by inserting a piece of thin glass between the sphere and each lever in turn. The method is exceedingly sensitive, but I have not been able to make it exact, owing to the warping produced in the rods due to unequal temperatures.

The measures of the expansion varied between ·0000214 and ·0000277, both vertical and horizontal diameters (in the position in the balance) being tested. The true value is probably nearly ·000025 or 1/40000. It will, therefore, lead to no appreciable error if we take the expansion as equal to that of the scale of the cathetometer, say 1/60000 (see p. 618, Table II.).

Determination of the Attraction by the Balance.

When the balance is used to measure such small forces and weights as those with which we are here concerned, it must be left swinging on its knife-edge throughout any set of weighings in which the deflections are to be compared one with another. For there is not the slightest reason to suppose that if the beam is lifted up and let down again, its new position of equilibrium will coincide with the old. And again, the beam, especially with such loads as the attracted masses, is put into a state of considerable strain, and continues to alter its shape sensibly for hours, and probably, even days, after the masses are put on to it. I have, therefore, always left the beam free for at least two or three days before commencing work with the balance, and it has of course remained free during the course of each day's work. The balance room was never entered just before any weighing, as it took many hours for the disturbance due to entrance and interference with the case to die away.

When the turn table supporting the attracting masses is moved half round, from one stop to the other, the bulk of the attraction is taken away from one attracted mass and put on to the other. The balance, being free, is slightly tilted over to the side on which is the larger attracting mass. But the deflection in the apparatus as arranged is so very small—at the most only 10 scale divisions—that errors of reading can only be neutralised by making a great number of successive measures.

Probably other errors are also largely eliminated, such as those due to the deposition of dust particles, shaking, change of ground level, and varying air currents. Of such errors I have found those due to varying air currents by far the worst. Sometimes—especially in autumn and winter—the balance will move quite irregularly through more than a scale division, and continue to move to and fro in this way for days or weeks. When in such an unsteady condition it is useless for accurate work. In spring and summer, however, it is much more steady as a rule, and frequently the scale can hardly be seen to move. I have never worked when on looking into the telescope for some time the irregular movements appeared to be more than a fraction of a tenth, *i.e.*, a fraction of one of the diagonal divisions, though, doubtless, irregularities comparable with a tenth of a whole division have often made their appearance in the work. It is perhaps not safe to ascribe these always to air currents.

I have always found the air steadiest in warm quiet weather, with a slowly rising temperature in the balance room, and most unsteady after a sudden fall of temperature. As the alteration of temperature spreads downwards, this is fully in accord with Lord RAYLEIGH'S observation that when the air is steady the ceiling is warmer than the floor, and that when it is unsteady the floor is the warmer of the two. In the observing room I had a gas stove often kept burning day and night, in the hope that the higher temperature it produced in the ceiling of the balance room below might steady the air. But the vertical walls of the balance room interfered with the action of the ceiling, and often produced unsteadiness.

A door opening or shutting anywhere in the building had a visible though transient effect, doubtless through an air wave. In a high wind the balance was always unsteady, partly, I suspect, through rushes of air into and out of the case with sudden pressure changes, and partly through changes of ground level, with variations of wind pressure against the building.

At all times there was a march in one direction or the other of the centre of swing. This was especially marked soon after the frame was lowered and the beam left free. As already remarked, readings were not taken till changes due to change in strain of the beam had subsided. But the march was very appreciable at other times, as will be seen from the diagrams. Perhaps the change was sometimes due to tilting of the ground, with barometric variation, since the balance was a very delicate level, and sometimes due to the change in buoyancy of the air affecting the two sides unequally, though I have not been able to make out any direct connection between barometric height and position of centre of swing. I believe that the explanation is to be sought for the most part in unsymmetrical effect on the beam of slight changes of temperature, for I have frequently noticed that a rising temperature produced an upward march, and a falling one a downward march. This explanation is supported by the following table of observations of the centre of swing, extending from May 9 to May 22, 1890, the balance being free, and the balance room undisturbed meanwhile.

The relation between temperature and centre of swing is represented in Diagram IX. (Plate 19.)

Date, 1890.	Time.	Centre of swing.	Temperature. Balance room.	Temperature. Observing room.	Barometer.
May 9 . .	11.5 A.M.	136·0	12°0	13°4	739·8
	12.55 P.M.	133·0	12·0	15·0	739·2
„ 12 . .	11.15 A.M.	133·8	12·05	14·5	738·6
	1.15 P.M.	131·9	12·05	15·8	738·3
	2.40 P.M.	133·7	12·05	16·6	738·1
Stove left on all night of 12th–13th.					
„ 13 . .	11.0 A.M.	181·7	12·6	17·5	740·2
	12.35 P.M.	181·5	12·6	18·4	740·3
	3.15 P.M.	185·0	12·7	18·6	740·3
Stove turned off.					
	5.25 P.M.	189·4	12·7	16·5	740·5
„ 14 . .	11.20 A.M.	167·4	12·6	14·3	745·8
	1.10 P.M.	165·5	12·6	14·4	746·0
„ 15 . .	11.5 A.M.	156·8	12·4	13·8	749·5
	2.45 P.M.	160·0	12·4	14·0	749·3
„ 16 . .	1.25 P.M.	158·3	12·4	14·0	744·3
„ 17 . .	8.50 P.M.	171·0	12·55	13·7	741·9
„ 19 . .	10.30 A.M.	174·3	12·55	14·0	743·5
	6.5 P.M.	181·8	12·6	14·0	741·0
„ 20 . .	11.30 A.M.	175·2	12·75	14·0	739·8
	1.5 P.M.	173·7	12·75	14·1	740·0
	5.20 P.M.	172·3	12·7	13·8	741·7
„ 21 . .	11.10 A.M.	172·7	12·6	13·7	751·9
„ 22 . .	11.30 A.M.	192 about	12·85	14·1	757·6

Of course, after a change in the position of the attracting masses or of the riders, the balance does not at once settle in a new position of equilibrium, but oscillates about it. Inasmuch as the balance never rests in this position, it is better to term it the centre of swing rather than the equilibrium position or resting point. The dashpot used to damp the vibrations of the mirror reflecting the scale serves also to damp those of the balance beam, and they die down rapidly. Instead of waiting, however, to observe directly the point on which they are closing in, it is much more exact, and also saves much time, to find the centre of swing as with an undamped balance from the extremities of the swings. I have always observed and recorded four extremities of three successive swings, occupying in all a little more than a minute.

Notwithstanding the very considerable damping, the successive lengths of swing are still in geometrical progression, but the rate of reduction is too great to allow the ordinary approximation, in which the geometrical is assumed to be an arithmetical progression. The exact method of determining the centre of swing is as follows :—

Let a, b, c, d be four successive readings of extremities of swing, and let x be the reading of the required centre.

Let the constant ratio of each swing length to the next be λ.

Then

$$a - x = \lambda(x - b) \quad \ldots \ldots \quad (1),$$

$$x - b = \lambda(c - x) \quad \ldots \ldots \quad (2),$$

$$c - x = \lambda(x - a) \quad \ldots \ldots \quad (3).$$

Eliminating λ from (1) and (2), we may readily obtain x in the form

$$x = c - \frac{(c-b)^2}{(a-b)+(c-b)} \quad \ldots \ldots \quad (4),$$

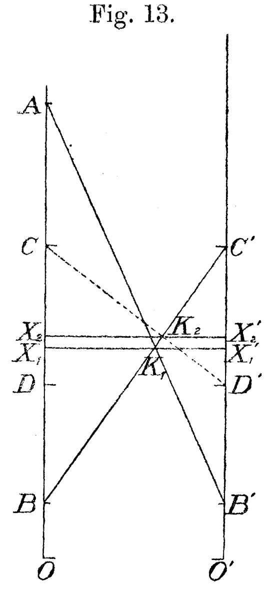
and from (2) and (3)

$$x = b + \frac{(c-b)^2}{(c-b)+(c-d)} \quad \ldots \ldots \quad (5).$$

With no disturbances and no errors of reading the values of x in (4) and (5) will coincide; but usually there is some small difference, the result of error or disturbance, and it is better to find both and take the mean. A third value might be obtained from (1) and (3); but it appears unadvisable to combine directly observations so far separated in time.

These formulæ lend themselves to easy arithmetical treatment, especially with the aid of a slide rule; but the following graphic method of finding the centre of swing is much less tiring and quite sufficiently exact.

Fig. 13.



Let the line OA, fig. 13, represent the scale; O its zero, and A, B, C, D the points distant respectively a, b, c, d from O.

Let O′C′ be a parallel line, B′, C′, D′ being points opposite to B, C, D respectively. Let AB′ and BC′ intersect in K_1. Draw $X_1K_1X_1'$ perpendicular to OA. Then X_1 is the centre of swing given by equation (4). For

$$\frac{AX_1}{X_1B} = \frac{AX_1}{X_1'B'} = \frac{K_1X_1}{K_1X_1'} = \frac{X_1B}{X_1'C'} = \frac{X_1B}{X_1C},$$

i.e., X_1 is the point dividing AB and BC in the same ratio. Similarly if BC′ and CD′ intersect in K_2, and $X_2K_2X_2'$ be drawn perpendicular to OA, X_2 is the point given by equation (5).

The third point given by equations (1) and (3) is obtained from the intersection of AB′ and CD′, but evidently a small error in C or D′ may considerably alter the position of this point, and it is better not to use it.

The construction was carried out thus: a large opal glass plate, 10″ × 11″, was etched with cross lines 10 to the inch, so as to present the appearance of ordinary section paper. The glaze was taken off so that pencil marks could be made. A diagonal line ran at 45° across the plate through the corners of the inch squares, and this was always taken as the line BC′ in the figure. Taking any convenient horizontal line, usually, of course, far below the plate, as zero, each inch represented a scale division, each tenth a diagonal division. The values of b and c fixed the lines to be taken as OA, O′C, and on these were marked the points A, C, B′, D′. A long glass slip, with a straight scratch on it, was then laid across from A to B′ so that the scratch passed through A and B′, and its intersection K_1 with the diagonal BC′, was x_1 from the zero line. The slip was then laid with the scratch passing through C and D′, and its intersection K_2 with BC′ gave x_2. It will be observed that all the actual construction for a set of readings of the balance swings consisted in marking four points on the plate.

The following cases, the first of very regular, the second of very disturbed swing, will serve to compare the results by this exact method with those obtained from the ordinary arithmetic mean method. At the same time they will show how nearly constant is the ratio of swing decrease.

Date and Number, 1890.	Scale readings in diagonal divisions.	Length of swing.	Ratio of each to preceding.	Centre of swing, exact.	Centre of swing, approximate $\frac{a+2b+c}{4}$.
May 4, No. 3	865 939 896 921	74 43 25	·581 ·581	911·8 911·8	909·75 913·0
				Mean 911·8	Mean 911·375
Sept. 17, No. 45	1118 1053 1093 1068	65 40 25	·615 ·625	1077·8 1078·6	1079·25 1076·75
				Mean 1078·2	Mean 1078·0

In finding the attraction the observations were always made in the same order, the determination of the scale value of rider and attraction being sandwiched so that each might be equally affected by any comparatively slow changes. Starting with the initial position, the attracting masses and riders were so arranged that, on moving either, the balance was deflected in the same direction and over the same part of the scale.

The following was the order of proceeding always observed, the column headed "Centre of Swing" being supposed to contain the values of the position in each case determined from four swing extremities as just explained :—

	Centre of swing.
(1) Initial position	i_1
(2) Riders moved	r_1
(3) Riders moved back to initial position . .	i_2
(4) Masses moved round	m_1
(5) Masses moved back to initial position . .	i_3
(6) Riders moved	r_2
(7) Riders moved back to initial position . .	i_4
(8) Masses moved round	m_2
and so on.	

To minimise the effect of progressive changes these observations were always combined in threes in the following way. Denoting the scale value of rider by R, and of attraction by M :—

From (1), (2), (3) . . . $R_1 = r_1 - \frac{i_1 + i_2}{2}$,

" (3), (4), (5) . . . $M_1 = m_1 - \frac{i_2 + i_3}{2}$,

" (5), (6), (7) . . . $R_2 = r_2 - \frac{i_3 + i_4}{2}$,

and so on.

These again were combined in threes, so that (the notation being continued) the successive values of attraction/rider are

$$\frac{2M_1}{R_1 + R_2}, \frac{M_1 + M_2}{2R_2}, \frac{2M_2}{R_2 + R_3}, \text{ and so on.}$$

The successive centres of swing i_1, r_1, i_2, m_1, &c., correspond to instants of time following each other at intervals of about 2 minutes, rather more than 1 minute being taken up in making and recording the four readings for each, and the rest in making the change of position in rider or mass and waiting for the next readings. It will be seen that each value of M or R is based on three successive centres of swing, the weighings extending over about 6 minutes, while each value of M/R is based on seven successive centres of swing determined in about 14 minutes.

A series of readings was usually continued for about 2 or 3 hours. The temperature in both observing and balance room was read at the beginning and end of the series, and the barometric height was also observed. As soon as possible after the desired number of determinations was completed with the attracted masses in one of the two positions, the vertical distances between attracting and attracted masses were measured by the cathetometer in the manner explained in Table II., and the position of the attracted masses was altered.

A full account of all the weighings is given in Table III., and the results are represented in Diagrams I.–VI. The three upper rows of points in each diagram represent the centres of swing, those in the initial position being marked • . After movement of the rider they are marked ×, and after movement of the masses they are marked ○. The base lines for the different rows are altered to save space, as described on the diagrams, for on the scale adopted the rider series would always be about 10 inches above or below the initial series. In Diagram I. the rider and mass series are also brought down and superposed on the initial series, so that each of the three has the same average height. It will be seen that all three are affected by the same disturbances. The advantage of the short time of swing and the mode of combining the results in threes will be realised more easily from this superposition.

The base line may be regarded as a time scale, as the instants corresponding to successive centres of swing were almost exactly equidistant.

In each case, under the representation of the centres of swing, are plotted the resulting values of M/R, and at the side will be found a representation of the distribution of results about the mean.

Assuming that each day's mean value is correct, and that the differences for different days are to be set down to variation of distance, &c., we can find the distribution of all the values about the mean by simply superposing the marginal curves at the side of the figures. The result fairly shows the accuracy as far as the weighing alone is concerned. It is represented in Diagram VII., where A is the mean value of the attraction in the lower, and a that in the upper position. A and a are brought near together to save space, but really they should be 40 inches apart. It will be seen that the range is about 2 per cent. of $A - a$ on each side of the mean, or taking the value of $A - a$ in milligrammes weight as about $\frac{1}{3}$ milligrm., and the load on each side as 20 kilogrms., the range is about $1/3 \times 10^9$ of this load on each side of the mean.

A comparison of the values of M/R in Diagrams I. and II., shows a very curious similarity in the fluctuations, and at first I was inclined to think there was some common external disturbance producing these fluctuations. But an analysis of the two sets of values appeared to show that the resemblance is merely accidental. When the values of M and R are set out separately, it is seen that the fluctuations depend chiefly on M, of which the fluctuations are slightly like each other for the two series, while those of R are quite different, but such that they make the fluctuations in M/R resemble each other much more closely than those in M alone. Further, it is not easy to see how fluctuations due to some external source would affect the values of M equally in the upper and lower positions and not have any effect on R. Some periodic change of level might be suspected, but this ought certainly to be traced in R. I have examined all the other diagrams and plotted out the component values of M and R, but have found no trace of resemblance, so that I think the curious likeness in I. and II. must be set down to accident.

There is a curious step by step descent of the centre of swing in the initial position on September 23, Diagram VI., which I cannot explain. It may be due to some error in the method of finding the centre of swing which comes in with a rapid march of that centre. The effect on the result is probably only small, for the value of M/R obtained with a march in the reverse direction on September 25 is very nearly the same, the two values being

September 23 . . . . ·2112753.
,, 25 . . . . ·2112533.

The following is a list of the weighings recorded, with the distances measured and the mean values of the attraction :—

SET I.

Date. 1890.	Position of attracted masses.	No. of values of M/R.	Mean values of M/R.	D or d in centims.	H or h in centims.
Feb. 4.	Upper . . .	50	·2142212	62·318	61·416
April 30 and May 4 .	Lower . . .	100	1·0109685	31·783	30·824
May 25	Upper . . .	50	·2157379	62·308	61·373

SET II.

Date. 1890.	Position of attracted masses.	No. of values of M/R.	Mean values of M/R.	D or d in centims.	H or h. in centims.
July 28	Lower . . .	25	·9973168	32·106	30·965
Sept. 17	Lower . . .	25	·9984148	32·116	30·954
Sept. 23 and 25 . .	Upper . . .	52	·2112647	62·708	61·566

On the completion of Set I. the four masses were inverted, and changed over from right to left or left to right, and the initial position was after this always arranged so that movement of rider or mass decreased the reading. This was done in order to lessen errors due to want of symmetry. If reversal had no effect, Set II. should, with the increased distance recorded above, give a value of M/R in the lower position of about ·990, instead of ·998. The larger value actually found is no doubt chiefly due to a want of symmetry in the large attracting mass M. The effect of this want of symmetry will be discussed after the investigation of the mathematical formula, and an account will be given of an independent method of detecting it. I think there is still outstanding a small difference, due, perhaps, to want of symmetry in the turn-table or in the attracted masses. The result of the reversal shows how necessary it was to make it. I should have liked to have in Set II. as many determinations as in Set I., so that the mean should be based on values of equal weight. During June and July, 1890, a complete set of 100 in each position, upper and lower, was made; but, owing to the pressure of other work, I was unable to calculate the results till the completion of the set. I then found that the value of M/R was still more than in Set II., and, on plotting out the results, it appeared that occasionally the rider value fell very considerably, and in an irregular way. On examination, there was little doubt that the rider came in contact occasionally with the suspending frame, when it was raised and should have been clear from it. Very likely temperature changes had brought about a displacement of the lever apparatus. Comparison with Set I. seemed to show that during that set no such contact had taken place, for there was no comparable irregularity. As it appeared dangerous to attempt to disentangle the good from the bad, the set of June and July was rejected, and Set II. was taken as recorded. When I had made the weighings giving 50 and 52 values in the two positions respectively, the balance became so irregular, through the cooler weather, that it was useless to continue work. Rather than carry over the experiment into another season, when it might be necessary to repeat the whole of the work, I have preferred to take Set II. as it stands, and give it the same weight as Set I. The final results are calculated from the means of Sets I. and II., as explained hereafter. I may here state the results obtained :—

$$\text{Constant of attraction} \quad . \quad . \quad G = \frac{6{\cdot}6984}{10^8}.$$

$$\text{Mean density of the Earth} \; . \quad \Delta = 5{\cdot}4934.$$

General Remarks on the Method.

Comparing the common balance with the torsion balance, there is no doubt that the former labours under the great disadvantage that the disturbances due to air currents are greatest in the vertical direction, that of the displacement to be measured. But even with this disadvantage the common balance may, I believe, be made to do much more than has hitherto been supposed possible. As an instrument in itself, apart from the external disturbances of air-currents, dust, &c., I believe its accuracy would be far beyond anything approached when these external disturbances are, as they always are, present to interfere with its action. I have always found that every precaution to ward off air currents and external disturbance has been accompanied by a corresponding increase in steadiness ; and I have seen no sign of a limit of accuracy depending on the instrument itself.

Besides the protection from air currents, there are two conditions essential above all others for accurate work :—

1st. That during any set of weighings in which the deflections are to be compared with each other, the beam should be supported on its knife-edge, and should be under constant strain.

2nd. That all moving parts, such as apparatus for changing riders or weights, should be supported quite independently of the balance or its case.

With regard to the first condition, it seems impossible to make the supporting frame move so truly and with so little disturbance that the knife edge shall return exactly to the same line. Even were it possible, the beam after raising and lowering would be practically a different beam, for, as my observations show, the condition of strain changes considerably after the load is first put on, and it would be merely a chance coincidence if the mean state of strain were the same during successive weighings. I have, in my former paper ('Proceedings of the Royal Society,' No. 190, 1878), described one method of comparing weights of nearly equal value with the beam throughout on its knife-edge and equally strained,* and I should now only modify that method in having regard to the second condition, of which I have since realised the importance when working with the large balance and with increased optical sensitiveness. It is surprising to find how much disturbance is produced by having the moving parts of the apparatus connected with the balance or its case.

As to air currents there is no doubt that, as Professor BOYS has shown, the greater

* I am glad that Dr. THIESEN urges the importance of this condition ('Travaux et Mémoires du Bureau International des Poids et Mesures,' vol. 5, " Études sur la Balance ").

the apparatus the greater the errors produced by them. At the time my apparatus was designed I did not know this, and there seemed to be a great advantage in making it large, as riders could be used of weight large enough to be measured accurately. Were I about to start with a new design I should certainly go towards the other extreme and make the apparatus small, attempting to get over the rider difficulty by some such method as that explained on p. 582. For not only is a smaller apparatus kept more easily at a uniform temperature, and, therefore, freer from the source of air currents, but it is much more handy to adjust, and even if the adjustments are not more accurate they will at least take much less time to make.

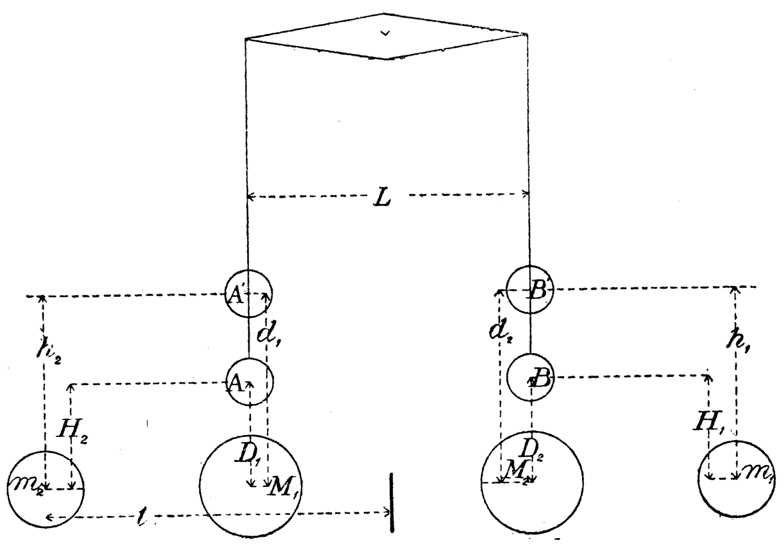
At the same time it is only fair to say, on behalf of the large apparatus, that some errors have been magnified on a like scale till they have become observable, and so could be investigated and eliminated. Starting with a small apparatus they would probably never have been detected, and would, therefore, have appeared in the final result.

II. Mathematical Investigation.

The Value of the Attraction Expressed in Terms of the Masses and Distances, and the Investigation of the Effect of Want of Symmetry in the Masses.

Let us suppose that initially the attracting masses are in the positions M_1, m_1, fig. 14, the larger on the left, the smaller on the right, and that the attracted masses

Fig. 14.



are in the lower position A, B. When the turntable is moved round so that the positions of the masses are M_2, m_2, the greater attraction is taken from the left and put on to the right. Let the centre of swing of the balance alter by an amount corresponding to a total change of vertical pull of n dynes. Assuming that a spherical mass M attracts another spherical mass M′ when their centres are D centimetres apart with a force of GMM'/D^2 dynes, we can express the change of vertical pull due to the change of position of the masses as G × a function F of the masses and distances. There is also a change of pull on the suspending rods and the balance beam which we may denote by E.

Then

$$n = GF + E.$$

In order to eliminate E let the attracted masses be moved into their upper positions A′, B′, and let the change on moving round the attracting masses be n' dynes. If f is the function of the masses and new distances corresponding to F,

$$n' = Gf + E.$$

Subtracting

$$n - n' = G(F - f),$$

whence

$$G = \frac{n - n'}{F - f},$$

and knowing G, the mean density of the earth may be at once found in the manner shown later.

We have then to find the form of the functions F, f, and as a preliminary step it is necessary to find the effect of the holes bored through the attracted masses A, B. This may be made to take the form of a correcting factor to the attraction which would be exercised on them if they were spheres.

The piece bored out in each case has radius ·31 centim. This we denote by c. It may be taken as practically a cylinder with plane ends and length equal to 15·8 centims., the diameter $2r$ of the spheres. The intensity due to such a cylinder of mass μ at D from its centre is (TODHUNTER'S 'An. Stat.,' Ed. 5, p. 292),

$$G\mu \frac{2r - \sqrt{\{(D + r)^2 + c^2\}} + \sqrt{\{(D - r)^2 + c^2\}}}{c^2 r},$$

which equals, to a sufficient approximation,

$$\frac{G\mu}{D^2 - r^2}.$$

If the mass remaining after μ is removed is A, and if the centre of the mass M is D below that of A, the attraction of M on A is

$$\frac{GM(A + \mu)}{D^2} - \frac{GM\mu}{D^2 - r^2}$$

$$= \frac{GMA}{D^2} + GM\mu\left(\frac{1}{D^2} - \frac{1}{D^2 - r^2}\right)$$

$$= \frac{GMA}{D^2}\left\{1 - \frac{\mu}{A}\left(\frac{r^2}{D^2} + \text{higher powers of } \frac{r^2}{D^2}\right)\right\}.$$

Now

$$\frac{\mu}{A} = \frac{2\pi c^2 r}{\frac{4}{3}\pi r^3 - 2\pi c^2 r} = \frac{3}{2}\frac{c^2}{r^2} \text{ nearly} = \frac{3}{2}\left(\frac{31}{790}\right)^2 = \cdot 00231,$$

and the greatest value of

$$\frac{r^2}{D^2} = \left(\frac{79}{320}\right)^2 = \cdot 061.$$

Then the higher powers may be neglected, and the attraction may be written

$$\frac{GMA}{D^2}\left(1 - \frac{3}{2}\frac{c^2}{D^2}\right) = \frac{GMA}{D^2}(1 - \theta), \text{ say.}$$

When A and B are in the lower position, $D = 32$, and $1 - \theta = \cdot 99986$. When they are in the upper position, $D = 62$ and $1 - \theta = \cdot 99996$, a value so near 1 that we shall in this position omit the correction, since it is only applied to one-fourth of the final result.

In the cross attractions we shall also omit the correction.

Referring to fig. 14 let the vertical differences of level between the centres of the various spheres be denoted as follows, the suffixes to M and m denoting their first and second positions respectively :—

$$\begin{array}{ll} A - M_1 = D_1 & B - M_1 = D_1', \\ B - M_2 = D_2 & A - M_2 = D_2', \\ B - m_1 = H_1 & A - m_1 = H_1', \\ A - m_2 = H_2 & B - m_2 = H_2'. \end{array}$$

When the masses A, B are placed in their upper positions, let the corresponding distances be denoted by small letters.

Let the horizontal distance between the centres of A and B be L, being within sensible limits equal to that between the centres of M in its two positions, and to the length of the beam, and let the radius of the circle in which m moves be l.

Then we have the following horizontal distances :—

$$\begin{aligned} A - M_2 &= B - M_1 = L, \\ A - m_1 &= B - m_2 = l + \tfrac{1}{2}L, \\ A - m_2 &= B - m_1 = l - \tfrac{1}{2}L. \end{aligned}$$

We may now write the change in vertical pull on the left by the motion of M from left to right, and of m from right to left, as follows—the first four terms representing the vertical attractions on A and B by M and m in their first position, the next four their attractions when moved round, and the last term E representing the change in attraction on the beam and suspending rods :—

$$G\left\{\frac{MA(1-\theta)}{D_1^2} - \frac{MBD_1'}{(D_1'^2+L^2)^{\frac{3}{2}}} - \frac{mBH_1}{\left\{H_1^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} + \frac{mAH_1'}{\left\{H_1'^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\right.$$
$$\left.+\frac{MB(1-\theta)}{D_2^2} - \frac{MAD_2'}{(D_2'^2+L^2)^{\frac{3}{2}}} - \frac{mAH_2}{\left\{H_2^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} + \frac{mBH_2'}{\left\{H_2'^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\right\} + E.$$

We may arrange all but the last term in nearly equal pairs.

Thus the first and fifth go together, and if we put $D_1 + D_2 = 2D$ and $D_1 + \delta = D_2 - \delta = D$, their sum is

$$GM(1-\theta)\left(\frac{A}{D_1^2}+\frac{B}{D_2^2}\right)$$
$$= GM(1-\theta)\left\{\frac{A}{D^2}\left(1+\frac{2\delta}{D}+\frac{3\delta^2}{D^2}+\ldots\right)+\frac{B}{D^2}\left(1-\frac{2\delta}{D}+\frac{3\delta^2}{D^2}-\ldots\right)\right\}$$
$$= GM(1-\theta)\frac{A+B}{D^2}\left\{\left(1+\frac{3\delta^2}{D^2}+\text{higher powers of}\frac{\delta^2}{D^2}\right)+\frac{A-B}{A+B}\cdot\frac{\delta}{D}\left(2+\frac{4\delta^2}{D^2}+\ldots\right)\right\}.$$

Now $(\delta/D)^2$ is negligible, as will be seen by reference to the table of distances, p. 617, and $(A-B)/(A+B)$ is less than $\frac{4}{3000}$, or of the same order as δ/D.

To a sufficiently close approximation then the sum of the two terms is

$$\frac{GM(A+B)(1-\theta)}{D^2}.$$

The second and sixth terms may also be taken together, and putting

$$D_1' + D_2' = 2D' \quad \text{and} \quad D_1' + \delta' = D_2' - \delta' = D',$$

we may show that to a sufficient approximation

$$GM\left\{\frac{BD_1'}{(D_1'^2+L^2)^{\frac{3}{2}}}+\frac{AD_2'}{(D_2'^2+L^2)^{\frac{3}{2}}}\right\} = \frac{GM(A+B)D'}{(D'^2+L^2)^{\frac{3}{2}}}.$$

The two pairs with m give similar results with

$$H = \tfrac{1}{2}(H_1 + H_2) \quad \text{and} \quad H' = \tfrac{1}{2}(H_1' + H_2').$$

Now

$$2D = D_1 + D_2 = A - M_1 + B - M_2$$
$$= B - M_1 + A - M_2 = D_1' + D_2' = 2D',$$

and similarly $2H = 2H'$, so that we may put the expression in the form

$$G\left\{\frac{M(A+B)(1-\theta)}{D^2} - \frac{M(A+B)D}{(D^2+L^2)^{\frac{3}{2}}} - \frac{m(A+B)H}{\left\{H^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} + \frac{m(A+B)H}{\left\{H^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\right\} + E = GF + E \text{ say.}$$

It is evident that we may combine experiments at different distances on different occasions in the same way by taking D and H to represent the mean values of these distances, so long as there is only a small variation from the mean.

If the attracted masses are now moved into their upper positions the expression for the change in attraction may be at once deduced from that in the lower position by replacing D and H by d and h, and omitting the factor $1-\theta$. Let it be denoted by $Gf + E$.

Subtracting one expression from the other E is eliminated, and we have

$$\begin{aligned} G(F-f) &= G\left\{\frac{M(A+B)(1-\theta)}{D^2} - \frac{M(A+B)D}{(D^2+L^2)^{\frac{3}{2}}} - \frac{m(A+B)H}{\left\{H^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} + \frac{m(A+B)H}{\left\{H^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\right. \\ &\quad \left. - \frac{M(A+B)}{d^2} + \frac{M(A+B)d}{(d^2+L^2)^{\frac{3}{2}}} + \frac{m(A+B)h}{\left\{h^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} - \frac{m(A+B)h}{\left\{h^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\right\}. \end{aligned}$$

This is to be equated to the difference in the values of the change in attraction in the two positions, as determined by the rider.

Let

b = the length of the small rider beam.
w = the mass of each rider.
A = mass deflection ÷ rider deflection in lower position.
a = ,, ,, ,, ,, ,, upper ,,
g_B = acceleration of gravity, or dynes weight per unit mass at Birmingham.

Then

$$G(F-f) = \frac{(A-a)\,bwg_B}{\frac{1}{2}L}.$$

Whence we may find the gravitation constant

$$G = \frac{2bwg_B(A-a)}{L(F-f)},$$

where all the quantities on the right hand are given in the Tables at the end.

The value of g_B may be found sufficiently nearly from the formula (EVERETT'S 'Units,' p. 21); $g_B = 980 \cdot 6056 - 2 \cdot 5028 \cos 2\lambda - \cdot 000003h$, where λ is the latitude $= 52° \, 28'$ at Birmingham, and h is the height above sea-level, which may be taken as 450 feet, or 13,725 centims. Whence $g_B = 981 \cdot 21$.

Since all the operations are conducted in air, the effective masses should throughout be less by the mass of air each displaces. But since they all have nearly the same densities, and w and $A + B$ appear respectively in numerator and denominator, it is sufficient to take their true masses, and to correct for air displaced in the case of M and m only.

To obtain the mean density of the earth Δ, we must express the acceleration of gravity in terms of G and the mass and dimensions of the earth.

The ordinary formula (PRATT, 'Figure of the Earth,' 4th ed., p. 119) is based on the assumption that the earth is a spheroid. It is sufficiently correct for our purpose, the departure of the assumed spheroid from the actual shape being very small. Adding a term $-3 \times 10^{-9}h$, or approximately, -41×10^{-6}, since the balance room is taken as 13,725 centims. above sea-level (see above), the value of gravity at Birmingham may be written

$$g_B = \frac{GV\Delta}{a^2}\left\{1 + \frac{\epsilon}{3} - \frac{3}{2}m + \left(\frac{5}{2}m - \epsilon\right)\sin^2 52° \, 28' - 41 \times 10^{-6}\right\},$$

where

V = volume of the earth $= 1 \cdot 0832 \times 10^{27}$ (EVERETT'S 'Units,' p. 57).

a = mean radius of the earth $= 6 \cdot 3709 \times 10^8$ (*loc. cit.*).

Δ = mean density of the earth.

m = equatorial "centrifugal force" $\div$ gravity $= \frac{1}{289}$.

ϵ = ellipticity of the earth $= \frac{1}{282}$.

The value of the ellipticity is taken to make the formula agree with that quoted above from EVERETT'S 'Units.' The uncertainty in the value is quite unimportant, for were ϵ as low as $\frac{1}{295}$, the error in Δ, introduced by taking it as $\frac{1}{282}$, would be less than 1 in 50,000.

Substituting for G, the value of the mean density of the earth is

$$\Delta = \frac{a^2 L (F - f)}{2bwV\left\{1 + \frac{\epsilon}{3} - \frac{3}{2}m + \left(\frac{5}{2}m - \epsilon\right)\sin^2 52° \, 28' - 41 \times 10^{-6}\right\}\left(A - a\right)}.$$

Here, as in the value of G, w and $A + B$ may have their true values, M and m their values less the mass of air displaced.

In the foregoing investigation we have supposed that all the masses are homogeneous and spherical, with the exception of the borings through A and B. We have supposed, also, that the turntable is exactly symmetrical about a vertical plane through its axis, so that its motion through two right angles is without effect. Doubtless, these suppositions and the formula based on them are not quite true. But, if we invert all the masses and change their sides, or pervert the whole arrangement of them, on taking the mean of the results obtained in the original and inverted and perverted positions we ought to greatly reduce the errors. Indeed, those due to want of symmetry in the turntable should evidently be quite eliminated, and those due to want of homogeneity in the masses should certainly be lessened.

To show this, we shall calculate the effect of a spherical "blow-hole," or gas cavity in M, in the first and most important term of F. This we shall take as being

$$\frac{\mathrm{GM}\,(\mathrm{A}+\mathrm{B})}{\mathrm{D}^2},$$

on the supposition that M is homogeneous and spherical.

If the mass of metal which would fill the blowhole is λ, supposing it to be placed there, the sphere is completed and its attraction is

$$\frac{\mathrm{G}\,(\mathrm{M}+\lambda)\,(\mathrm{A}+\mathrm{B})}{\mathrm{D}^2};$$

but the vertical attraction is less than this in reality by the vertical component of the attraction of λ.

Let

B be the centre of the cavity,
P the centre of the attracted mass,
O the centre of the attracting mass,
δ the distance of B from the centre of M,
θ the angle BOP.

The vertical component of the attraction of λ is

$$\frac{\mathrm{G}\lambda\,(\mathrm{A}+\mathrm{B})\cos\mathrm{BPO}}{\mathrm{PB}^2},$$

but

$$\mathrm{BP}^2 = \mathrm{D}^2 + \delta^2 - 2\mathrm{D}\delta\cos\theta,$$

and

$$\cos\mathrm{BPO} = \frac{\mathrm{D}-\delta\cos\theta}{\mathrm{BP}},$$

whence the attraction of λ may be put

$$\frac{\mathrm{G}\lambda\,(\mathrm{A}+\mathrm{B})\,(\mathrm{D}-\delta\cos\theta)}{(\mathrm{D}^2+\delta^2-2\mathrm{D}\delta\cos\theta)^{\frac{3}{2}}} = -\,\mathrm{G}\lambda\,(\mathrm{A}+\mathrm{B})\,\frac{d}{d\mathrm{D}}\,\frac{1}{\sqrt{\mathrm{D}^2+\delta^2-2\mathrm{D}\delta\cos\theta}},$$

$$= \frac{\mathrm{G}\lambda\,(\mathrm{A}+\mathrm{B})}{\mathrm{D}^2}\left(1 + 2\mathrm{P}_1\frac{\delta}{\mathrm{D}} + 3\mathrm{P}_2\frac{\delta^2}{\mathrm{D}^2} + \ldots\right),$$

where P_1, P_2 . . . are zonal harmonics. The attraction of the sphere with the cavity is therefore

$$\frac{\mathrm{GM}\,(\mathrm{A}+\mathrm{B})}{\mathrm{D}^2}\left\{1 - \frac{\lambda}{\mathrm{M}}\left(2\mathrm{P}_1\frac{\delta}{\mathrm{D}} + 3\mathrm{P}_2\frac{\delta^2}{\mathrm{D}^2} + 4\mathrm{P}_3\frac{\delta^3}{\mathrm{D}^3} + \ldots\right)\right\}.$$

If the mass is inverted, the vertical component is obtained by changing the sign of δ, and the mean of the two values is

$$\frac{\text{GM}(\text{A}+\text{B})}{\text{D}^2}\left\{1-\frac{\lambda}{\text{M}}\left(3\text{P}_2\frac{\delta^2}{\text{D}^2}+5\text{P}_4\frac{\delta^4}{\text{D}^4}+\dots\right)\right\},$$

the first power of δ/D being eliminated.

If $\theta = 0$, P_2 and all the other harmonics $= 1$.

If $\theta = 90°$, $\text{P}_2 = -\frac{1}{2}$, $\text{P}_4 = \frac{3}{8}$, &c.

Now, with the actual dimensions of the apparatus, $(\delta/\text{D})^2$ cannot be so great as $(\frac{1}{2})^2$ or $\frac{1}{4}$, and may, of course, be much smaller. The first term of those involving λ, therefore, is the most important, and it lies between $+\frac{3}{2}(\lambda/\text{M})(\delta^2/\text{D}^2)$ and $-3(\lambda/\text{M})(\delta^2/\text{D}^2)$ changing sign for the value of θ given by $\text{P}_2 = 0$.

The second set of experiments recorded in this paper was taken after inversion and change of side of all the masses, and the final result obtained from this set differs by a little more than 1 per cent. from that obtained from the first set, the observed attraction being slightly greater at the same distance. The difference may be due to irregularities in any or all of the masses and in the turntable, and to other undetected effects, such as change of level on rotating the turntable. It would be a very long task to disentangle these, and I have contented myself with trying to find how much must be set down to irregularity in the large mass M, by taking a set of weighings with it alone inverted.

After the weighings on July 28, and the subsequent measures of distances, M was inverted only, and the other masses remained as in Set II. Some weeks later on, September 14, 25 values of $\text{M}/\text{R} = \text{A}$ were obtained, the mean being $\cdot 9926$. The distances were $\text{D} = 32\cdot 118$, $\text{H} = 30\cdot 978$. The mass M was then put in its original position, as in Set II., and on September 17, on referring to the tables, it will be seen that the value of M/R obtained was $\cdot 9984$, the distances being $\text{D} = 32\cdot 117$ and $\text{H} = 30\cdot 955$, practically the same as on September 14.

Assuming that the difference in attraction is due to cavities in various places, and that, for each, the term $3\text{P}_2\delta^2/\text{D}^2$ is negligible, we have, approximately,

$$\frac{1-2\dfrac{\Sigma\lambda\text{P}_1\delta}{\text{MD}}}{1+2\dfrac{\Sigma\lambda\text{P}_1\delta}{\text{MD}}}=\frac{9926}{9984}.$$

Whence, approximately, since $\text{D} = 32$,

$$\frac{\Sigma\lambda\text{P}_1\delta}{\text{M}} = \cdot 0464 \text{ centim.}$$

This result may be tested by independent experiment. For, let the centre of gravity be x below the horizontal plane through the point bisecting the vertical diameter (*i.e.*, the centre of figure), in the position of September 14. The distance of any missing particle λ from the horizontal plane is $\delta \cos \theta = P_1\delta$. Completing the sphere by the addition of all such particles, the centre of gravity is brought to the centre of figure, so that we have

$$Mx = \Sigma\lambda P_1\delta,$$

and

$$x = \frac{\Sigma\lambda P_1\delta}{D}.$$

We have, therefore, to determine the vertical distance of the centre of gravity from the centre of figure.

In order to do this, a large flat-bottomed scale-pan (one belonging to the balance used in the gravitation experiment) was suspended by two parallel wires about 8 centims. apart and 3 metres long. In the middle of the pan was a shallow cup about 7·5 centims. internal diameter, arranged so that it could turn freely but truly about a vertical axis. The mass, M, was placed on this cup with the diameter, which had been vertical, arranged horizontal, and perpendicular to the plane of the suspending wires. A vertical flat plate, worked by a horizontal micrometer screw, could be brought just in contact with the end of the diameter, and the reading of the micrometer gave the position of the point of contact. The position of the scale-pan was determined by a plumb line hanging over one edge in front of a horizontal scale. On turning the cup and mass through 180°, and repeating the readings, knowing the weight of the scale-pan and the position of its centre of gravity, x could at once be found.

Two separate experiments gave

$$x = \cdot 0536 \text{ centim.},$$

and

$$x = \cdot 0516 \text{ centim.},$$

not very different from the value ·0464, obtained from the attraction experiments. The agreement is, I think, very close when it is noted that a difference in the attraction in one of the sets of weighings of 1 in 1000 would make x either ·038 or ·054.

This result appears to justify the rejection of all terms in the expansion above the first, and so supports the belief that the reversal largely eliminates errors due to irregularity of shape. For it is in the case of M that there is the greatest danger of large value for δ/D, and the above experiments seem to indicate that even in this case it is small.

It is, perhaps, noteworthy that the largest term rejected in the attraction of M, viz., $3\lambda P_2\delta^2/MD^2$ is, if we give P_2 its maximum value 1,

$$\frac{3\lambda\delta}{MD} \cdot \frac{\delta}{D} = \frac{3x}{D} \cdot \frac{\delta}{D},$$

which is not greater than

$$\frac{3 \times \cdot 0464}{32} \times \frac{15}{32} = \cdot 0020,$$

since the radius of the mass is 15.

This is in a term about 5/4 of the final result, so that the greatest error which can be introduced by neglecting this term is ·0025, or 1 in 400.

In calculating the results of the experiments the means of Sets I. and II. have been taken. Equal weights have been given to each set. It would have been more satisfactory if the number of experiments had been the same in each set; but I should have had to wait for another season to obtain more, and then it would, probably, have been necessary to repeat the whole series in both arrangements, as it is not safe to assume that the various disturbing causes remain the same over a wide interval of time. The second set, though fewer in number, are, in some respects, I believe, better; partly owing to the additional experience gained when they were taken.

In order that the various terms in $F - f$ may be compared, I give below their numerical values, as determined from the values of the masses and distances given in the tables. The meaning of each term in the first column will be seen on referring to fig. 14. The second column contains the actual values; the third column the values in terms of the fourth, the lowest term.

VALUE of $F - f$.

$\dfrac{M(A+B)(1-\theta)}{D^2}$	$= 6483938 \cdot 8$	416
$-\dfrac{M(A+B)D}{(D^2+L^2)^{\frac{3}{2}}}$	$-\ \ 102416 \cdot 3$	6·6
$-\dfrac{m(A+B)H}{\left\{H^2+\left(l-\dfrac{L}{2}\right)^2\right\}^{\frac{3}{2}}}$	$-\ \ 316243 \cdot 3$	20
$+\dfrac{m(A+B)H}{\left\{H^2+\left(l+\dfrac{L}{2}\right)^2\right\}^{\frac{3}{2}}}$	$+\ \ 15579 \cdot 9$	1
$-\dfrac{M(A+B)}{d^2}$	$-\ 1693687 \cdot 2$	109
$+\dfrac{M(A+B)d}{(d^2+L^2)^{\frac{3}{2}}}$	$+\ \ 156728 \cdot 0$	10
$+\dfrac{m(A+B)h}{\left\{h^2+\left(l-\dfrac{L}{2}\right)^2\right\}^{\frac{3}{2}}}$	$+\ \ 310695 \cdot 0$	20
$-\dfrac{m(A+B)h}{\left\{h^2+\left(l+\dfrac{L}{2}\right)^2\right\}^{\frac{3}{2}}}$	$-\ \ 27597 \cdot 7$	1·7

Whence $F - f = 4826997 \cdot 2$.

The mean value of $A - a$ (see Table III.), is

$$A - a = \cdot 791295;$$

substituting these values of $F - f$ and $A - a$ in the formula for G (p. 606), we obtain

$$G = \frac{6 \cdot 6984}{10^8};$$

substituting them in the formula for Δ we obtain

$$\Delta = 5 \cdot 4934.$$

The values given by Sets I. and II., treated separately, are to two figures of decimals,

Set I.	$\Delta = 5 \cdot 52$
Set II.	$\Delta = 5 \cdot 46.$

III.—TABLES.

TABLE I.—Constants of the Apparatus and Dimensions of the Earth.

Masses.

		grms.
Attracting mass M, in vacuo	=	153407·26
Less air displaced, say	=	153388·85
Attracting mass m, in vacuo	=	76497·4
Less air displaced, say	=	76488·2
Attracted mass A, in vacuo	=	21582·33
,, ,, B, ,,	=	21566·21
,, ,, A + B, in vacuo	=	43148·54
Riders each, in vacuo	=	0·010119

Vertical Diameters of Masses in terms of Cathetometer Scale correct at 18°.

The masses are taken as having the same coefficient of expansion as the scale.

		centims.
M	=	30·526
m	=	24·176
A	=	15·8203
B	=	15·7829.

The diameters of the masses A and B are taken between the nuts securing them on the suspending wires.

		centims.
Balance beam at 0°, L	=	123·232
Rider beam at 0°, b	=	2·53575
L/b (as occurring explicitly in G and Δ, independent of temperature, assuming them to have the same coefficient of expansion)	=	48·59775

Latitude of Birmingham	=	52° 28′
Height of balance room above sea-level	=	13725 centims.
Gravity at Birmingham, g_B	=	981·21 centims./sec.^{2}
Mean radius of earth	=	$6·3709 \times 10^8$ centims.
Volume of earth	=	$1·0832 \times 10^{27}$ cub. centims.
Equatorial "centrifugal force"/gravity	=	$\frac{1}{289}$
Ellipticity of Earth	=	$\frac{1}{283}$

$$1 + \frac{\epsilon}{3} - \frac{3m}{2} + \left(\frac{5}{2}m - 2\right)\sin^2 52^\circ\, 28' - 41 \times 10^{-6} . = \cdot 999161.$$

TABLE II.—Vertical and Horizontal Distances.

Vertical Diameters of Masses taken by the Cathetometer, described p. 588.

In the tables below P.S. signifies divisions on the scale over which moves the pointer, which is attached to the small adjustment plate. V.S. signifies divisions on the vertical millimetre scale.

DIAMETER of Large Attracting Mass M.

	Reading on pointer scale.	Mean.
Upper telescope sighting top of mass . .	73·2, 73·4, 73·2	73·27 P.S.
Lower ,, ,, bottom ,, . .	23·0, 23·0, 23·2	23·07 P.S.

TURNING round to the Vertical Scale.

	Reading on pointer scale.	Mean.
Upper telescope sighting 459 millims. V.S.	94·6, 94·9, 94·4, 95·0, 94·0	94·58 P.S.
,, ,, ,, 458 ,,	68·8, 68·8, 69·7, 69·0, 68·7, 70·0, 69·6, 69·4	69·25 P.S.

$$\text{Therefore } 25{\cdot}33 \text{ P.S. divisions} = 1 \text{ millim. V.S.,}$$

$$\text{and scale reading for top of mass} = 458 + \frac{73{\cdot}27 - 69{\cdot}25}{25{\cdot}33}$$

$$= 458{\cdot}158 \text{ millims. V.S.}$$

	Reading on pointer scale.	Mean.
Lower telescope sighting 153 millims. V.S.	27·3, 27·4, 27·7, 27·0	27·35 P.S.
,, ,, ,, 152 ,,	0·0, 0·3, − 0·5, 0·0	− 0·07 P.S.

$$\text{Therefore } 27{\cdot}42 \text{ P.S. divisions} = 1 \text{ millim. V.S.,}$$

$$\text{and scale reading for bottom of mass} = 152 + \frac{23{\cdot}07 + 0{\cdot}07}{27{\cdot}42}$$

$$= 152{\cdot}844 \text{ millims. V.S.}$$

$$\text{The difference} = 30{\cdot}5314 \text{ centims.}$$

This is rather greater than the diameter of the mass, as the cross wire was made to touch the image of the mass in each case. A series of measures of 1 millim. on the scale, in which the cross wire was on the centre of each division, and of 1 millim. between the jaws of a wire gauge, in which the wire touched the images of the jaws, showed that at the distance at which the scale was, ·005 centim. must be subtracted, leaving

Diameter of M = 30·526 centims.

VERTICAL Diameter of Small Attracting Mass m.

	Reading on pointer scale.	Mean.
Upper telescope sighting top of mass . .	75·6, 75·6, 75·0	75·40 P.S.
Lower ,, ,, bottom ,, . .	26·5, 26·3, 26·8	26·53 P.S.

TURNING round to the Vertical Scale.

	Reading on pointer scale.	Mean.
Upper telescope sighting 388 millims. V.S.	100, 99·9, 99·7	99·87 P.S.
,, ,, ,, 387 ,,	73·9, 73·4, 74·0	73·77 P.S.

Therefore 26·10 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for top of mass} = 387 + \frac{75{\cdot}40 - 73{\cdot}77}{26{\cdot}10}$$

$$= 387{\cdot}062 \text{ millims. V.S.}$$

	Reading on pointer scale.	Mean.
Lower telescope sighting 146 millims. V.S.	45·9, 45·9, 45·0	45·60 P.S.
,, ,, ,, 145 ,,	20·4, 19·4, 20·0	19·93 P.S.

Therefore 25·77 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for bottom of mass} = 145 + \frac{26{\cdot}53 - 19{\cdot}93}{25{\cdot}77}$$

$$= 145{\cdot}256 \text{ millims. V.S.}$$

The difference = 24·1806 centims.

Subtracting the same correction as in the last case for the cross wire,

Diameter of m = 24·176 centims.

Vertical Diameters of Attracted Masses A and B taken between the Junctions of the Securing Nuts with the Sphere.

A.

	Reading on pointer scale.	Mean.
Upper telescope sighting top of mass . .	82·7, 83·0, 82·9	82·9 P.S.
Lower " " bottom " . .	31·0, 31·5, 31·3	31·3 P.S.

TURNING round to the Vertical Scale.

	Reading on pointer scale.	Mean.
Upper telescope sighting 429 millims. V.S.	95·0, 95·2, 95·3	95·3 P.S.
" " " 428 "	69·0, 69·0, 68·9	69·0 P.S.

Therefore 26·3 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for top of mass} = 428 + \frac{82·9 - 69·0}{26·3}$$

$$= 428·531 \text{ millims. V.S.}$$

	Reading on pointer scale.	Mean.
Lower telescope sighting 271 millims. V.S.	48·0, 47·8, 48·4	48·1 P.S.
" " " 270 "	23·4, 23·0, 22·8	23·1 P.S.

Therefore 25·0 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for bottom of mass} = 270 + \frac{31·3 - 23·1}{25·0}$$

$$= 270·328 \text{ millims. V.S.}$$

The difference gives the diameter since the middle of the cross wire was used, so that

Diameter of A = 15·8203 centims.

B.

	Reading on pointer scale.	Mean.
Upper telescope sighting top of mass . .	72·0, 71·0, 71·0	71·3 P.S.
Lower " " bottom " . .	24·6, 25·0, 25·2	24·9 P.S.

TURNING round to the Vertical Scale.

	Reading on pointer scale.	Mean.
Upper telescope sighting 430 millims. V.S.	94·0, 94·6, 94·0	94·2 P.S.
,, ,, ,, 429 ,,	68·0, 68·1, 68·1	68·1 P.S.

$$\text{Therefore } 26{\cdot}1 \text{ P.S. divisions} = 1 \text{ millim. V.S.,}$$

$$\text{and scale reading for top of mass} = 429 + \frac{71{\cdot}3 - 68{\cdot}1}{26{\cdot}1}$$

$$= 429{\cdot}123 \text{ millims. V.S.}$$

	Reading on pointer scale.	Mean.
Lower telescope sighting 272 millims. V.S.	43·3, 43·0, 43·0	43·1 P.S.
,, ,, ,, 271 ,,	17·1, 17·3, 17·4	17·3 P.S.

$$\text{Therefore } 25{\cdot}8 \text{ P.S. divisions} = 1 \text{ millim. V.S.,}$$

$$\text{and scale reading for bottom of mass} = 271 + \frac{24{\cdot}9 - 17{\cdot}3}{25{\cdot}8}$$

$$= 271{\cdot}294 \text{ millims. V.S.}$$

$$\text{And diameter of B} = 15{\cdot}7829 \text{ centims.}$$

Vertical Distances between the Levels of the Centres of the Attracting and Attracted Masses Measured by Cathetometer.

The measurements were made as soon as possible after the completion of a set of weighings, usually on the following day.

It was necessary to fix the attracted masses in the position occupied during the weighings, and with the beam of the balance in the same strained condition. This was done in some cases by gripping the left suspending wire by a pair of jaws; in others, by adding a small weight to one side, and placing a block of the right thickness under the mass on that side.

The cathetometer was placed in front of the left side of the balance case, from which position all the masses could be viewed by turning the telescope round the central pillar (fig. 2). It was read when sighting the top of each attracting mass and the top of each attracted mass when in the lower position, the bottom of each when in the upper position, the top and bottom being taken at the junctions of the securing nuts with the masses. It is therefore necessary to add to the distances measured by the cathetometer the difference of the radii of attracting and attracted masses in the lower position, and their sum in the upper position (see p. 613). The work is shown in full for February 5 and May 5.

Tests were made at various times, showing that there was no change in the distances (at least within errors of reading), either through moving the turn-table or in the course of a few days (see February 5 and May 5 for examples).

Temperature Correction.—The cathetometer scale is taken as correct at 18°, and its coefficient of expansion is assumed to be 1/60000. That of the masses is probably about 1/40000, but, for simplicity, is taken as equal to that of the scale, the difference, 1/120000, never amounting to as much as the errors of reading, since the greatest length concerned is 23 centims.

The temperature was estimated to be about 1° above that observed during the immediately preceding weighings, the presence of the observer and the lights used tending to raise it.

The cathetometer rested always on the brick floor of the room. Its vernier reads to ·002 centim.

Set I.

Attracted masses A on the left, B on the right. Attracting mass M moving round from left to right in front of the balance case.

February 5, 1890.—Attracted masses in upper position. Assumed temperature 11°.

Half way through the measurements the cathetometer was accidentally moved, and could not be exactly replaced. Repeating the reading of A it was found that ·197 must be added to the previous readings to compare with the following ones. This addition is made where the numbers have an asterisk.

	A. 64·999* 65·001*	B. 65·284 65·282	
m_2 23·448	M_1 25·895* 25·889*	M_2 26·070 26·064	m_1 23·947*
Differences : $A - m_2 = 41·552$	$A - M_1 = 39·108$	$B - M_2 = 39·216$	$B - m_1 = 41·336$

From Table I., p. 613, the sums of the radii of the masses are

$$R_M + R_A = 23·173,$$
$$R_M + R_B = 23·154,$$
$$R_m + R_A = 19·998,$$
$$R_m + R_B = 19·979,$$

whence

$$d = \tfrac{1}{2}\{39{\cdot}108 + 23{\cdot}173 + 39{\cdot}216 + 23{\cdot}154\}$$
$$= 62{\cdot}326,$$

and

$$h = \tfrac{1}{2}\{41{\cdot}336 + 19{\cdot}979 + 41{\cdot}552 + 19{\cdot}998\}$$
$$= 61{\cdot}433.$$

These are in terms of a scale correct at 18°, so that the value is too great by about 7/60000. We take as true values

Corrected

$$d = 62{\cdot}318,$$
$$h = 61{\cdot}425.$$

Test experiment.—At the conclusion, the distance $A - M_1$ was measured again and found to be 39·110.

May 28, 1890.—Attracted masses in upper position. Assumed temperature 14°.

	A.	B.	
	64·674 64·674	65·286 65·288	
m_2	M_1	M_2	m_1
23·422 23·424	25·726 25·724	25·920 25·920	23·766 23·756
Differences: $A - m_2 = 41{\cdot}552$	$A - M_1 = 38{\cdot}949$	$B - M_2 = 39{\cdot}367$	$B - m_1 = 41{\cdot}526$

whence

$$d = 62{\cdot}312,$$
$$h = 61{\cdot}377.$$

Subtracting temperature correction ·004,

Corrected

$$d = 62{\cdot}308,$$
$$h = 61{\cdot}373.$$

Mean values in Set I.,

$$d = 62{\cdot}313,$$
$$h = 61{\cdot}399.$$

May 5, 1890.—Attracted masses in lower position. Assumed temperature 13°.

	A.	B.	
	50·324 50·324 50·328	50·622 50·634 50·630	
m_2	M_1	M_2	m_1
23·672 23·674	25·972 25·970 25·972	26·138 26·138 26·132	23·998 24·008
Differences: $A - m_2 = 26·652$	$A - M_1 = 24·354$	$B - M_2 = 24·493$	$B - m_1 = 26·626$

From Table I., p. 613,

$$R_M - R_A = 7·353,$$
$$R_M - R_B = 7·372,$$
$$R_m - R_A = 4·178,$$
$$R_m - R_B = 4·197,$$

whence

$$D = \tfrac{1}{2}\{24·354 + 7·353 + 24·493 + 7·372\}$$
$$= 31·786,$$

and

$$H = \tfrac{1}{2}\{26·626 + 4·197 + 26·652 + 4·178\}$$
$$= 30·827.$$

Subtracting temperature correction ·0025,

Corrected values for Set I.,

$$D = 31·783,$$
$$H = 30·824.$$

Test Experiment.—The balance was set free at the end of these measures, and two days later, on May 7, it was again fixed, and the distance D was determined by the cathetometer described on p. 588. The value obtained was D = 31·786.

NOTE.—If the apparatus were perfectly rigid and constant in its dimensions we should expect $D - H = d - h =$ constant. The values actually given by the above experiments are

February 5 . . . . ·892,
May 5 . . . . ·959,
May 28 . . . . ·935.

There is apparently a slight increase during the course of the spring, probably due to the warping of the wood supporting the mass m. But there was some uncertainty in sighting the top of the mass m, especially when in the distant position on the right.

Set II.

Attracted masses A on the right, B on the left. Attracting mass M moving round from left to right behind the balance case. All the masses inverted.

July 29, 1890.—Attracted masses in lower position. Assumed temperature 16°.

	B. 49·014 49·014	A. 49·846 49·844	
m_1 22·434 22·436	M_2 24·584 24·586	M_1 24·788 24·782	m_2 22·868 22·864
Differences: $B - m_1 = 26·579$	$B - M_2 = 24·429$	$A - M_1 = 25·060$	$A - m_2 = 26·979$

whence

$$D = 32·107,$$
$$H = 30·967.$$

Subtracting temperature correction ·001,

Corrected

$$D = 32·106,$$
$$H = 30·966.$$

September 18, 1890.—Attracted masses in lower position. Assumed temperature 16°.

	B. 49·076 49·074	A. 49·768 49·766	
m_1 22·467	M_2 24·576 24·576	M_1 24·756 24·758	m_2 22·840
Differences: $B - m_1 = 26·608$	$B - M_2 = 24·499$	$A - M_1 = 25·010$	$A - m_2 = 26·927$

whence

$$D = 32·117,$$
$$H = 30·955.$$

Subtracting temperature correction ·001, *Corrected*

$$D = 32{\cdot}116,$$
$$H = 30{\cdot}954.$$

Mean values in Set II.,

$$D = 32{\cdot}111,$$
$$H = 30{\cdot}960.$$

September 27, 1890.—Attracted masses in upper position. Assumed temperature 16°.

	B. 63·880 63·876	A. 64·540 64·544	
m_1 22·450 22·448	M_2 24·570 24·572	M_1 24·756 24·758	m_2 22·810 22·816
Differences : $B - m_1 = 41{\cdot}429$	$B - M_2 = 39{\cdot}307$	$A - M_1 = 39{\cdot}785$	$A - m_2 = 41{\cdot}729$

whence

$$d = 62{\cdot}710,$$
$$h = 61{\cdot}568.$$

Subtracting temperature correction ·002,

Corrected values for Set II.,

$$d = 62{\cdot}708,$$
$$h = 61{\cdot}566.$$

NOTE.—The values of $D - H$ and $d - h$, which should be constant, are from the above, and from another set of measures (not here recorded, see p. 609) on September 15, as follows. (We have no reason to expect the same value as in Set I., as the masses M, m, have changed sides.)

July 29	1·140,
September 15	1·140,
September 18	1·162,
September 27	1·142.

From July 29 to September 15 inclusive, the balance was swinging freely without alteration. The values of H should, therefore, be the same on those dates. They were

July 29	30·967,
September 15	30·978,

equal almost within errors of reading for the top of m.

Means of Sets I. and II. :—

$$D = \tfrac{1}{2}(31{\cdot}783 + 32{\cdot}111)$$
$$= 31{\cdot}947.$$
$$H = \tfrac{1}{2}(30{\cdot}824 + 30{\cdot}960)$$
$$= 30{\cdot}892.$$
$$d = \tfrac{1}{2}(62{\cdot}313 + 62{\cdot}708)$$
$$= 62{\cdot}511.$$
$$h = \tfrac{1}{2}(61{\cdot}399 + 61{\cdot}566)$$
$$= 61{\cdot}483.$$

Horizontal Distances.

SET I.

$$L = 123{\cdot}269 \text{ centims.}$$

At 18°

$$l_1 = 122{\cdot}915 \quad ,,$$

and

$$\frac{L}{2} = 61{\cdot}635 \quad ,,$$

whence

$$l_1 + \frac{L}{2} = 184{\cdot}550 \quad ,,$$

$$l_1 - \frac{L}{2} = 61{\cdot}280 \quad ,,$$

Taking the mean temperature of the Set as 12°, and assuming 1/60000 as the coefficient of expansion, on correcting to 12°,

$$l_1 + \frac{L}{2} = 184{\cdot}532 \text{ centims.}$$

$$l_1 - \frac{L}{2} = 61{\cdot}274 \quad ,,$$

SET II.

At 18°

$$l_2 = 122{\cdot}795 \text{ centims.}$$

$$\frac{L}{2} = 61{\cdot}635 \quad ,,$$

Whence

$$l_2 + \frac{L}{2} = 184{\cdot}430 \quad ,,$$

$$l_2 - \frac{L}{2} = 61{\cdot}160 \quad ,,$$

Taking the mean temperature of the Set as 15°, and correcting to 15°,

$$l_2 + \frac{L}{2} = 184{\cdot}421 \text{ centims.}$$

$$l_2 - \frac{L}{2} = 61{\cdot}157 \quad ,,$$

Mean values for the two Sets

$$L = 123{\cdot}260 \quad ,,$$

$$l + \frac{L}{2} = 184{\cdot}477 \quad ,,$$

$$l - \frac{L}{2} = 61{\cdot}216 \quad ,,$$

TABLE III.—DETERMINATION OF ATTRACTION BY THE BALANCE.

Determinations of the Attraction in terms of the Riders by the Balance.

In each case four turning points of three successive swings are recorded in tenths of a division, *i.e.*, in divisions on the diagonal lines. In the columns headed i the masses and riders are in the initial position, in those headed r the riders are moved, and in those headed m the masses are moved. Under each set of four readings is the calculated centre of swing (see p. 595). In the next line are the deflections due to movements of riders and masses, each placed under the middle one of the three centres of swing from which it is calculated. In the next line are the values of deflection due to mass ÷ deflection due to rider, or M/R (see p. 598).

SET I.

I. ATTRACTED Masses in Upper Position. Feb. 4, 1890, 7.59 P.M. to 10.49 P.M. Temperature: in Observing Room, 15°·7–16°·5; in Balance Room, 10°·05. Barometer 752·2–752·0 millims. Weather mild and still, after slight frost on the two previous nights. Time between successive passages of centre about 20 seconds.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings	725 798 759 780	912 838 878 856	725 799 759 781	804 787 796 791	764 779 771 776	913 838 879 857	726 800 759 781	804 787 797 791
Centre of swing . . .	772·55	863·85	773·00	792·80	773·90	864·60	773·40	793·20
Deflection due to rider or mass	..	91·075	..	19·350	..	90·950	..	19·700
Mass deflection ÷ rider deflection	..	..	..	·212608	..	·214688	..	·217110

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	763 779 771 775	913 837 879 857	724 801 759 782	804 787 796 792	764 779 771 776	914 838 880 857	725 801 760 783	805 789 796 792
Centre of swing . . .	773·60	864·25	773·85	793·05	773·90	865·05	774·50	793·65
Deflection due to rider or mass	..	90·525	..	19·175	..	90·850	..	18·950
Mass deflection ÷ rider deflection	..	·214720	..	·211440	..	·209824	..	·208758

TABLE III. (continued)

	i. (17)	r. (18)	i. (19)	m. (20)	i. (21)	r. (22)	i. (23)	m. (24)
Scale readings	765 780 772 777	916 839 881 859	726 803 762 784	803 788 797 792	765 779 771 776	914 838 879 857	725 800 759 782	805 787 796 791
Centre of swing	774·90	866·30	776·30	793·65	774·00	864·50	773·60	792·85
Deflection due to rider or mass	..	90·700	..	18·500	..	90·700	..	19·225
Mass deflection ÷ rider deflection	..	·206174	..	·203966	..	·207966	..	·211438

	i. (25)	r. (26)	i. (27)	m. (28)	i. (29)	r. (30)	i. (31)	m. (32)
Scale readings	763 780 770 776	914 838 880 857	727 800 760 782	805 789 797 792	764 779 771 775	913 838 879 857	725 800 760 781	804 786 796 791
Centre of swing . . .	773·65	865·05	774·15	793·90	773·65	864·65	773·80	792·50
Deflection due to rider or mass	..	91·150	..	20·000	..	90·925	..	19·275
Mass deflection ÷ rider deflection	..	·215167	..	·219690	..	·215975	..	·211494

	i. (33)	r. (34)	i. (35)	m. (36)	i. (37)	r. (38)	i. (39)	m. (40)
Scale readings	763 778 770 774	913 838 879 857	724 801 759 782	803 789 796 792	763 778 769 774	912 837 879 857	726 799 759 780	802 787 797 792
Centre of swing . . .	772·65	864·60	773·85	793·50	772·30	864·20	772·95	793·30
Deflection due to rider or mass	..	91·350	..	20·425	..	91·575	..	19·875
Mass deflection ÷ rider deflection	..	·217296	..	·223316	..	·220038	..	·216503

TABLE III. (continued).

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings . . .	764 779 771 776	915 839 880 858	725 800 759 780	803 786 795 791	763 778 770 774	913 838 879 857	726 799 759 781	803 787 796 792
Centre of swing . . .	773·90	86·555	773·15	792·25	772·70	864·60	773·15	792·95
Deflection due to rider or mass	. .	92·025	. .	19·325	. .	91·675	. .	19·200
Mass deflection ÷ rider deflection	. .	·212986	. .	·210397	. .	·210117	. .	·209693

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)	*m.* (56)
Scale readings	764 779 772 776	914 839 880 858	724 802 759 781	803 787 795 790	762 778 769 774	912 836 877 855	721 798 755 779	801 785 794 788
Centre of swing . . .	774·35	865·55	773·85	792·10	772·20	862·55	770·35	790·55
Deflection due to rider or mass	. .	91·450	. .	19·075	. .	91·275	. .	20·275
Mass deflection ÷ rider deflection	. .	·209267	. .	·208784	. .	·215557	. .	·222161

	i. (57)	*r.* (58)	*i.* (59)	*m.* (60)	*i.* (61)	*r.* (62)	*i.* (63)*	*i.* (63*a*)	*m.* (64)
Scale readings .	760 776 767 772	911 836 877 855	724 799 758 779	803 785 795 789	762 777 769 773	911 836 877 855	722 800 758 780	725 799 759 780	803 786 796 790
Centre of swing	770·20	862·50	772·20	791·35	771·70	862·50	772·50	772·90	792·30
Deflection due to rider or mass . . .	. .	91·250	. .	19·425	. .	90·400	. .	. .	19·900
Mass deflection ÷ rider deflection . .	. .	·217534	. .	·213873	. .	·217506	. .	. .	·217873

* After 63 the riders were moved by mistake instead of the masses, therefore it was necessary to return to the initial position, and take the readings in (63*a*).

TABLE III. (continued).

	i. (65)	*r.* (66)	*i.* (67)	*m.* (68)	*i.* (69)	*r.* (70)	*i.* (71)	*m.* (72)
Scale readings	762 777 769 774	913 838 879 857	725 800 758 780	802 785 794 789	759 776 767 772	909 834 875 854	722 797 757 779	802 784 794 789
Centre of swing . . .	771·90	864·60	772·75	790·80	770·15	860·80	771·05	790·55
Deflection due to rider or mass	. .	92·275	. .	19·350	. .	90·200	. .	19·450
Mass deflection ÷ rider deflection	. .	·213170	. .	·212084	. .	·215078	. .	·214947

	i. (73)	*r.* (74)	*i.* (75)	*m.* (76)	*i.* (77)	*r.* (78)	*i.* (79)	*m.* (80)
Scale readings	760 777 768 773	911 835 877 854	724 798 757 779	803 785 795 790	762 777 769 773	911 835 877 854	721 797 756 778	800 784 793 788
Centre of swing . . .	771·15	862·05	771·40	791·50	771·70	862·05	770·30	789·75
Deflection due to rider or mass	. .	90·775	. .	19·950	. .	91·050	. .	20·150
Mass deflection ÷ rider deflection	. .	·217020	. .	·219442	. .	·220209	. .	·221064

	i. (81)	*r.* (82)	*i.* (83)	*m.* (84)	*i.* (85)	*r.* (86)	*i.* (87)	*m.* (88)
Scale readings . . .	759 774 766 771	910 833 876 854	722 796 757 778	801 783 793 787	759 775 767 771	910 834 876 854	723 797 757 778	802 783 793 787
Centre of swing . . .	768·90	860·95	770·50	789·30	769·60	861·25	770·90	789·35
Deflection due to rider or mass	. .	91·250	. .	19·250	. .	91·000	. .	19·300
Mass deflection ÷ rider deflection	. .	·215990	. .	·211248	. .	·211813	. .	·212995

TABLE III. (continued).

	i. (89)	*r.* (90)	*i.* (91)	*m.* (92)	*i.* (93)	*r.* (94)	*i.* (95)	*m.* (96)
Scale readings	759 775 766 771	908 831 874 852	719 795 754 776	800 783 792 788	760 775 767 772	910 835 876 854	723 798 756 779	800 785 793 789
Centre of swing . . .	769·20	859·00	768·35	789·05	769·90	861·60	770·90	790·30
Deflection due to rider or mass	. .	90·225	. .	19·925	. .	91·200	. .	19·350
Mass deflection ÷ rider deflection	. .	·217373	. .	·219650	. .	·215323	. .	·212462

	i. (97)	*r.* (98)	*i.* (99)	*m.* (100)	*i.* (101)	*r.* (102)	*i.* (103)	*m.* (104)	*i.* (105)
Scale readings .	761 777 768 772	910 835 876 853	721 796 756 777	798 783 791 787	759 775 765 770	909 833 874 852	721 796 756 777	800 783 793 787	759 775 767 771
Centre of swing	771·00	861·35	769·80	788·35	768·60	859·65	769·80	789·35	769·60
Deflection due to rider or mass . . .	. .	90·950	. .	19·150	. .	90·450	. .	19·650	
Mass deflection ÷ rider deflection . .	. .	·211655	. .	·211136	. .	·214483			

Feb. 4, 1890. Mean of 50 determinations of M/R = a } ·21422122.
Attracted masses in upper position }

TABLE III. (continued).

II.—ATTRACTED Masses in Lower Position. April 30, 1890, 7.45 P.M. to 10.32 P.M. Temperature: in Observing Room, 17°–16°·1; in Balance Room, 11°·1; Barometer, 748·6–749·2 millims. Weather clear; S.E. wind; sunny during day. Time between successive passages of centre not quite 20 seconds.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings	1046 969 1012 988	1133 1055 1098 1075	951 1024 984 1007	1127 1062 1099 1078	955 1025 986 1007	1134 1059 1102 1077	952 1028 985 1009	1123 1069 1099 1082
Centre of swing	996·60	1082·85	998·35	1085·50	999·80	1086·25	1000·40	1088·25
Deflection due to rider or mass	..	85·375	..	86·425	..	86·150	..	87·350
Mass deflection ÷ rider deflection	..	..	..	1·00772	..	1·00856	..	1·01437

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	962 1023 989 1009	1136 1060 1104 1079	955 1029 987 1012	1129 1067 1102 1083	961 1027 989 1012	1136 1064 1105 1081	956 1031 989 1013	1133 1069 1103 1085
Centre of swing	1001·40	1088·00	1002·45	1089·55	1003·15	1090·00	1004·15	1091·25
Deflection due to rider or mass	..	86·075	..	86·750	..	86·350	..	86·350
Mass deflection ÷ rider deflection	..	1·01133	..	1·00623	..	1·00232	..	1·00101

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings	967 1027 994 1012	1141 1064 1108 1083	957 1034 990 1015	1135 1070 1106 1086	965 1031 994 1015	1143 1066 1110 1085	958 1036 993 1017	1134 1073 1108 1089
Centre of swing	1005·65	1092·00	1006·00	1093·20	1007·40	1094·00	1008·35	1095·40
Deflection due to rider or mass	..	86·175	..	86·500	..	86·125	..	86·825
Mass deflection ÷ rider deflection	..	1·00290	..	1·00406	..	1·00624	..	1·01106

TABLE III. (continued).

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings	976 1027 998 1016	1143 1069 1110 1087	963 1037 996 1019	1141 1073 1112 1090	966 1037 996 1019	1145 1070 1112 1089	960 1040 995 1020	1141 1075 1113 1092
Centre of swing . . .	1008·80	1095·35	1010·65	1097·90	1010·85	1097·05	1011·15	1099·30
Deflection due to rider or mass	..	85·625	..	87·150	..	86·050	..	87·575
Mass deflection ÷ rider deflection	..	1·01591	..	1·01529	..	1·01264	..	1·01713

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings	973 1034 1000 1020	1147 1071 1114 1090	962 1041 996 1022	1137 1079 1112 1094	975 1034 1002 1020	1146 1075 1114 1091	965 1042 999 1021	1138 1080 1113 1094
Centre of swing . . .	1012·30	1098·55	1012·50	1100·20	1013·40	1099·80	1014·00	1101·00
Deflection due to rider or mass	..	86·150	..	87·250	..	86·100	..	86·650
Mass deflection ÷ rider deflection	..	1·01465	..	1·01306	..	1·00987	..	1·00858

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings	977 1035 1003 1022	1150 1073 1116 1093	964 1043 1000 1025	1089 1110 1098 1104	976 1038 1004 1023	1153 1074 1118 1094	968 1044 1001 1025	1148 1078 1118 1095
Centre of swing . . .	1014·70	1100·80	1015·45	1102·20	1016·15	1102·35	1016·45	1103·40
Deflection due to rider or mass	..	85·725	..	86·400	..	86·050	..	86·475
Mass deflection ÷ rider deflection	..	1·00933	..	1·00597	..	1·00450	..	1·00625

TABLE III. (continued).

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)	*m.* (56)
Scale readings	978 1039 1005 1025	1153 1076 1119 1094	969 1045 1002 1027	1149 1080 1118 1097	976 1043 1005 1026	1153 1079 1121 1096	968 1047 1004 1028	1145 1085 1118 1099
Centre of swing . . .	1017·40	1103·35	1017·65	1104·45	1018·60	1105·55	1019·25	1106·15
Deflection due to rider or mass	..	85·825	..	86·325	..	86·625	..	86·875
Mass deflection ÷ rider deflection	..	1·00670	..	1·00116	..	·99971	..	1·00973

	i. (57)	*r.* (58)	*i.* (59)	*m.* (60)	*i.* (61)	*r.* (62)	*i.* (63)	*m.* (64)
Scale readings	984 1039 1008 1026	1155 1078 1120 1097	971 1048 1004 1029	1151 1083 1122 1100	977 1046 1007 1030	1157 1081 1123 1100	972 1051 1007 1031	1152 1087 1123 1102
Centre of swing . . .	1019·30	1105·10	1020·00	1107·85	1021·25	1108·10	1022·60	1109·95
Deflection due to rider or mass	..	85·450	..	87·225	..	86·175	..	86·850
Mass deflection ÷ rider deflection	..	1·01872	..	1·01646	..	1·01011	..	1·00798

	i. (65)	*r.* (66)	*i.* (67)	*m.* (68)	*i.* (69)	*r.* (70)	*i.* (71)	*m.* (72)
Scale readings	983 1046 1011 1031	1159 1082 1125 1102	976 1051 1008 1033	1153 1088 1126 1104	983 1049 1012 1032	1161 1083 1127 1102	978 1053 1011 1034	1158 1088 1127 1106
Centre of swing . . .	1023·60	1109·80	1023·70	1112·05	1025·15	1111·05	1025·95	1113·15
Deflection due to rider or mass	..	86·150	..	87·625	..	85·500	..	86·700
Mass deflection ÷ rider deflection	..	1·01262	..	1·02097	..	1·01944	..	1·01226

TABLE III. (continued).

	i. (73)	*r.* (74)	*i.* (75)	*m.* (76)	*i.* (77)	*r.* (78)	*i.* (79)	*m.* (80)
Scale readings	985 1051 1014 1033	1163 1086 1129 1104	980 1056 1013 1036	1156 1092 1129 1108	990 1051 1017 1036	1163 1087 1131 1107	982 1057 1015 1039	1161 1093 1132 1109
Centre of swing . . .	1026·95	1113·40	1028·25	1115·50	1029·20	1115·20	1030·15	1117·65
Deflection due to rider or mass	..	85·880	..	86·775	..	85·525	..	87·100
Mass deflection ÷ rider deflection	..	1·01093	..	1·01299	..	1·01652	..	1·01782

	i. (81)	*r.* (82)	*i.* (83)	*m.* (84)	*i.* (85)	*r.* (86)	*i.* (87)	*m.* (88)
Scale readings	991 1054 1018 1038	1167 1090 1133 1108	984 1059 1018 1041	1158 1098 1132 1112	992 1056 1021 1041	1169 1092 1136 1111	984 1063 1018 1042	1161 1100 1135 1115
Centre of swing . . .	1030·95	1117·40	1032·60	1119·55	1033·55	1120·00	1034·00	1122·20
Deflection due to rider or mass	..	85·625	..	86·475	..	86·225	..	87·600
Mass deflection ÷ rider deflection	..	1·01358	..	1·00640	..	1·00942	..	1·01890

	i. (89)	*r.* (90)	*i.* (91)	*m.* (92)	*i.* (93)	*r.* (94)	*i.* (95)	*m.* (96)
Scale readings	996 1058 1022 1043	1171 1094 1137 1114	987 1064 1022 1045	1165 1100 1137 1117	995 1061 1024 1045	1172 1097 1137 1116	989 1066 1023 1046	1169 1099 1140 1117
Centre of swing . . .	1035·20	1121·75	1036·85	1123·75	1037·35	1123·15	1038·20	1125·05
Deflection due to rider or mass	..	85·725	..	86·650	..	85·375	..	86·575
Mass deflection ÷ rider deflection	..	1·01633	..	1·01286	..	1·01450	..	1·01065

TABLE III. (continued).

	i. (97)	*r.* (98)	*i.* (99)	*m.* (100)	*i.* (101)	*r.* (102)	*i.* (103)	*m.* (104)	*i.* (105)
Scale readings	998 1062 1026 1045	1175 1097 1141 1116	992 1066 1025 1047	1174 1102 1141 1119	995 1064 1027 1048	1176 1098 1143 1118	995 1067 1026 1049	1169 1105 1143 1121	1001 1066 1029 1049
Centre of swing	1038·75	1125·05	1039·45	1127·15	1040·15	1126·70	1040·75	1128·90	1042·20
Deflection due to rider or mass . . .	..	85·950	..	87·350	..	86·250	..	87·425	
Mass deflection ÷ rider deflection . .	..	1·01178	..	1·01452	..	1·01319			

April 30. Mean of 50 determinations of M/R = A } 1·010905.
Attracted masses in lower position

MAY 4, 1890, 11.11 to 11.50 A.M. Temperature: in Observing Room, 13°·5 to 13°·8; in Balance Room, 11°·7; Barometer, 742·0 to 741·7 millims. Weather inclined to rain; a little cooler than previous day; wind S. to S.W.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings	875 936 900 920	1045 969 1013 988	865 939 896 921	1044 970 1014 989	865 938 897 920	1045 967 1013 986	861 940 894 921	1041 971 1012 989
Centre of swing . . .	913·10	996·95	911·80	997·75	911·60	996·00	910·95	997·10
Deflection due to rider or mass	..	84·500	..	86·050	..	84·725	..	86·275
Mass deflection ÷ rider deflection	..	..	..	1·01699	..	1·01697	..	1·01950

TABLE III. (continued).

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	869 936 896 919	1046 966 1012 985	862 939 894 920	1036 972 1009 988	867 936 896 918	1045 964 1011 984	860 937 893 919	1040 968 1009 986
Centre of swing . . .	910·700	995·10	910·45	995·55	910·40	993·80	909·20	994·20
Deflection due to rider or mass	..	84·525	..	85·125	..	84·000	..	85·450
Mass deflection ÷ rider deflection	..	1·01390	..	1·01024	..	1·01533	..	1·01454

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)	*i.* (25)
Scale readings	863 934 894 916	1043 964 1009 983	859 936 892 917	1033 971 1006 985	863 932 892 915	1040 963 1007 982	857 936 889 915	1035 966 1006 983	862 930 891 914
Centre of swing	908·30	992·60	908·00	993·15	906·60	991·00	906·10	991·40	905·25
Deflection due to rider or mass . . .	..	84·450	..	85·850	..	84·650	..	85·725	
Mass deflection ÷ rider deflection . .	..	1·01421	..	1·01538	..	1·01344			

May 4. Morning. Mean of 10 determinations of $M/R = A$ } 1·015050.
Attracted masses in lower position

TABLE III. (continued).

MAY 4, 1890.—*Same day.* 2.40 to 4.54 P.M. Temperature: in Observing Room, 13°·9–14°·1; in Balance Room, 11°·7–11°·75; Barometer, 740·3–739·7.

	i. (1)	r. (2)	i. (3)	m. (4)	i. (5)	r. (6)	i. (7)	m. (8)
Scale readings . . .	847 933 883 912	1035 957 1003 977	853 930 886 911	1031 961 1002 979	864 925 890 909	1035 960 1003 977	853 931 885 912	1026 965 1001 980
Centre of swing . . .	901·40	986·20	902·00	987·05	902·55	987·10	902·00	987·65
Deflection due to rider or mass	..	84·500	..	84·775	..	84·825	..	85·300
Mass deflection ÷ rider deflection	..	..	..	1·00133	..	1·00251	..	1·00783

	i. (9)	r. (10)	i. (11)	m. (12)	i. (13)	r. (14)	i. (15)	m. (16)
Scale readings	866 924 891 909	1035 959 1004 977	853 932 886 912	1023 968 1000 982	864 925 889 910	1036 960 1004 978	854 931 886 912	1024 968 1000 982
Centre of swing . . .	902·70	987·20	902·80	988·35	902·30	987·75	902·50	988·45
Deflection due to rider or mass	..	84·450	..	85·800	..	85·350	..	85·925
Mass deflection ÷ rider deflection	..	1·01303	..	1·01060	..	1·00601	..	1·00940

	i. (17)	r. (18)	i. (19)	m. (20)	i. (21)	r. (22)	i. (23)	m. (24)
Scale readings . . .	864 925 890 909	1039 958 1005 978	855 931 887 913	1025 969 1001 982	866 925 891 911	1040 958 1005 978	855 932 888 913	1011 977 996 985
Centre of swing . . .	902·55	987·80	903·25	989·20	903·50	987·90	904·0	989·10
Deflection due to rider or mass	..	84·900	..	85·825	..	84·150	..	85·225
Mass deflection ÷ rider deflection	..	1·01148	..	1·01538	..	1·01634	..	1·01232

TABLE III. (continued).

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings . . .	864 927 890 912	1036 961 1004 979	854 933 887 914	1024 970 1001 984	865 926 892 912	1037 962 1005 980	854 934 888 915	1031 967 1004 983
Centre of swing . . .	903·75	988·10	904·00	989·85	904·35	989·25	904·90	990·55
Deflection due to rider or mass	..	84·225	..	85·675	..	84·625	..	85·625
Mass deflection ÷ rider deflection	..	1·01454	..	1·01481	..	1·01211	..	1·01182

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings	864 928 892 912	1039 961 1006 980	855 934 888 914	1024 972 1002 985	864 929 891 913	1041 961 1006 980	856 934 888 915	1025 971 1002 984
Centre of swing . . .	904·95	989·50	904·80	991·10	905·00	989·65	905·00	990·65
Deflection due to rider or mass	..	84·625	..	86·200	..	84·650	..	85·500
Mass deflection ÷ rider deflection	..	1·01521	..	1·01846	..	1·01418	..	1·00796

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings	866 927 893 913	1038 962 1007 981	857 934 889 915	1030 960 1004 984	865 930 893 915	1043 962 1008 981	858 934 891 915	1035 966 1006 984
Centre of swing . . .	905·30	990·40	905·50	991·30	906·60	991·15	906·55	991·55
Deflection due to rider or mass	..	85·000	..	85·250	..	84·575	..	84·95
Mass deflection ÷ rider deflection	..	1·00441	..	1·00546	..	1·00621	..	1·00741

TABLE III. (continued).

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)	*m.* (56)
Scale readings	869 927 895 914	1041 961 1008 982	858 935 891 916	1037 965 1007 984	867 929 894 914	1041 963 1007 982	856 936 890 916	1029 972 1004 986
Centre of swing . . .	906·65	990·90	907·00	991·80	906·60	991·05	906·70	992·50
Deflection due to rider or mass	..	84·075	..	85·000	..	84·440	..	85·750
Mass deflection ÷ rider deflection	..	1·01070	..	1·00905	..	1·01155	..	1·01509

	i. (57)	*r.* (58)	*i.* (59)	*m.* (60)	*i.* (61)	*r.* (62)	*i.* (63)	*m.* (64)
Scale readings	871 928 895 913	1041 963 1008 982	859 936 890 917	1035 969 1008 985	869 931 895 916	1044 963 1010 983	860 937 892 917	1030 973 1005 986
Centre of swing . . .	906·80	991·50	907·10	993·50	908·20	992·85	908·25	993·30
Deflection due to rider or mass	..	84·550	..	85·850	..	84·625	..	84·775
Mass deflection ÷ rider deflection	..	1·01478	..	1·01493	..	1·00812	..	1·00162

	i. (65)	*r.* (66)	*i.* (67)	*m.* (68)	*i.* (69)	*r.* (70)	*i.* (71)	*m.* (72)
Scale readings	863 935 894 917	1042 965 1010 984	863 935 894 917	1039 969 1008 985	840 949 886 923	1042 965 1009 985	861 937 893 918	1037 971 1009 987
Centre of swing . . .	908·80	993·45	908·80	993·75	909·15	993·25	909·05	995·05
Deflection due to rider or mass	..	84·650	..	84·775	..	84·150	..	85·750
Mass deflection ÷ rider deflection	..	1·00148	..	1·00444	..	1·01322	..	1·01449

TABLE III. (continued).

	i. (73)	*r.* (74)	*i.* (75)	*m.* (76)	*i.* (77)	*r.* (78)	*i.* (79)	*m.* (80)
Scale readings	865 935 895 918	1045 965 1011 985	860 938 893 919	1038 969 1010 987	868 934 895 918	1045 965 1012 985	863 937 894 919	1041 969 1011 988
Centre of swing . . .	909·55	994·30	909·25	995·00	909·50	994·75	909·80	995·80
Deflection due to rider or mass	..	84·900	..	85·625	..	85·100	..	85·625
Mass deflection ÷ rider deflection	..	1·00928	..	1·00735	..	1·00617	..	1·00765

	i. (81)	*r.* (82)	*i.* (83)	*m.* (84)	*i.* (85)
Scale readings	864 938 895 919	1044 967 1012 986	860 940 894 920	1036 974 1010 989	867 936 896 919
Centre of swing . . .	910·55	995·45	910·65	996·80	910·60
Deflection due to rider or mass	..	84·850	..	86·175	
Mass deflection ÷ rider deflection	..	1·01238			

May 4, afternoon. Mean of 40 determinations of M/R = A, Attracted masses in lower position: 1·0100278.

April 30 and May 4. Mean of 100 determinations of M/R = A, Attracted masses in upper position: 1·0109685.

TABLE III. (continued).

III.—ATTRACTED Masses in Upper Position. May 25, 1890; 11.20 to 12.53 noon; Temperature: in Observing Room, 15°·4–16°; in Balance Room, 13°·3; Barometer, 748·5–748·1 millims. Weather, E. wind, warm, very bright. Time of swing not recorded.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings	1071 986 1033 1005	1175 1085 1134 1108	960 1047 998 1025	1049 1028 1041 1034	1003 1021 1010 1017	1173 1085 1134 1107	956 1047 996 1024	1049 1028 1040 1034
Centre of swing . . .	1015·90	1116·90	1015·50	1036·20	1014·25	1116·55	1014·25	1035·80
Deflection due to rider or mass	..	101·200	..	21·325	..	102·300	..	21·500
Mass deflection ÷ rider deflection	..	..	..	·209582	..	·209311	..	·210320

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	1001 1021 1011 1016	1173 1085 1134 1106	958 1046 996 1024	1049 1028 1038 1033	1002 1020 1010 1015	1173 1084 1134 1105	957 1046 995 1023	1049 1028 1040 1032
Centre of swing . . .	1014·35	1116·35	1014·05	1034·70	1013·50	1115·80	1013·25	1035·90
Deflection due to rider or mass	..	102·150	..	20·925	..	102·425	..	22·850
Mass deflection ÷ rider deflection	..	·207660	..	·204571	..	·213688	..	·223297

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings	1003 1019 1009 1015	1173 1082 1133 1104	956 1043 994 1023	1048 1025 1038 1032	1001 1019 1008 1014	1172 1081 1131 1103	958 1042 994 1021	1048 1026 1038 1030
Centre of swing . . .	1012·85	1114·60	1011·90	1033·60	1012·05	1113·10	1011·35	1033·50
Deflection due to rider or mass	..	102·225	..	21·625	..	101·400	..	22·375
Mass deflection ÷ rider deflection	..	·217535	..	·212400	..	·216962	..	·220172

TABLE III. (continued).

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings	1000 1017 1007 1014	1171 1081 1130 1103	953 1044 992 1021	1047 1025 1037 1031	1000 1016 1007 1013	1171 1080 1130 1103	955 1041 992 1021	1046 1025 1036 1030
Centre of swing . . .	1010·90	1112·70	1010·80	1032·90	1010·45	1112·40	1010·00	1032·15
Deflection due to rider or mass	..	101·850	..	22·275	..	102·175	..	22 050
Mass deflection ÷ rider deflection	..	·219195	..	·218356	..	·216907	..	·215885

	i. (33)	*r.* (34)	*i* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings	999 1017 1006 1013	1168 1080 1131 1102	952 1043 992 1021	1046 1024 1037 1030	999 1018 1007 1012	1170 1082 1130 1102	955 1043 993 1021	1046 1026 1039 1030
Centre of swing . . .	1010·20	1112·40	1010·40	1032·35	1010·70	1112·65	1011·05	1033·75
Deflection due to rider or mass	..	102·100	..	21·800	..	101·775	..	22·100
Mass deflection ÷ rider deflection	..	·214740	..	·213857	..	·215672	..	·216858

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings	998 1019 1009 1014	1171 1082 1132 1104	955 1043 994 1022	1046 1027 1038 1031	1000 1019 1009 1014	1173 1082 1131 1104	956 1046 995 1023	1048 1028 1039 1032
Centre of swing . . .	1012·25	1114·00	1011·65	1033·85	1012·35	1113·80	1013·20	1034·85
Deflection due to rider or mass	..	102·050	..	21·850	..	101·025	..	21·900
Mass deflection ÷ rider deflection	..	·215388	..	·215197	..	·216531	..	·216350

TABLE III. (continued).

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings	1001 1019 1009 1015	1172 1083 1132 1105	956 1046 996 1023	1048 1027 1039 1032	1000 1020 1009 1016	1173 1082 1133 1105	956 1044 995 1023
Centre of swing . . .	1012·70	1114·60	1013·65	1034·60	1013·15	1114·70	1012·65
Deflection due to rider or mass	..	101·425	..	21·200	..	101·800	
Mass deflection ÷ rider deflection	..	·212472	..	·208636			

May 25. Morning. Mean of 25 determinations of $M/R = a$ Attracted masses in upper position } ·21446168.

Same Day. 3.15 to 4.50 P.M. Temperature : in Observing Room, 16°·0 to 16°·25 ; in Balance Room, 13°·3 to 13°·35 ; Barometer, 747·7–747·4 millims.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings	1001 1069 1031 1052	1205 1116 1165 1138	990 1077 1029 1057	1081 1061 1073 1066	1034 1055 1044 1049	1207 1120 1168 1139	991 1080 1030 1059	1083 1062 1076 1068
Centre of swing . . .	1044·55	1147·60	1046·30	1068·55	1047·60	1150·50	1048·35	1070·65
Deflection due to rider or mass	..	102·175	..	21·600	..	102·525	..	21·675
Mass deflection ÷ rider deflection	..	..	..	·211041	..	·211046	..	·212162

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	1037 1056 1046 1052	1209 1119 1169 1141	995 1082 1030 1059	1086 1066 1078 1071	1039 1058 1048 1054	1213 1121 1172 1145	994 1086 1034 1064	1088 1069 1078 1073
Centre of swing . . .	1049·60	1151·10	1049·00	1073·50	1051·60	1154·10	1052·90	1074·90
Deflection due to rider or mass	..	101·800	..	23·200	..	101·850	..	21·675
Mass deflection ÷ rider deflection	..	·220408	..	·227842	..	·220299	..	·216738

TABLE III. (continued).

	i. (17)	r. (18)	i. (19)	m. (20)	i. (21)	r. (22)	i. (23)	m. (24)
Scale readings	1045 1059 1050 1056	1212 1126 1175 1147	996 1088 1037 1066	1088 1071 1080 1076	1043 1063 1053 1058	1215 1126 1177 1148	1002 1088 1039 1066	1094 1072 1083 1077
Centre of swing . . .	1053·55	1157·20	1055·30	1077·10	1056·30	1158·50	1056·55	1079·25
Deflection due to rider or mass	..	102·775	..	21·300	..	102·075	..	22·350
Mass deflection ÷ rider deflection	..	·209073	..	·207957	..	·213813	..	·219575

	i. (25)	r. (26)	i. (27)	m. (28)	i. (29)	r. (30)	i. (31)	m. (32)
Scale readings	1044 1064 1053 1061	1217 1127 1178 1150	999 1093 1041 1069	1095 1073 1086 1079	1048 1067 1056 1061	1221 1131 1180 1152	1002 1094 1042 1071	1096 1076 1088 1081
Centre of swing . . .	1057·25	1159·80	1059·35	1081·35	1059·70	1162·50	1060·70	1083·55
Deflection due to rider or mass	..	101·500	..	21·825	..	102·300	..	22·800
Mass deflection ÷ rider deflection	..	·217611	..	·214181	..	·218112	..	·223147

	i. (33)	r. (34)	i. (35)	m. (36)	i. (37)	r. (38)	i. (39)	m. (40)
Scale readings	1049 1067 1057 1064	1221 1131 1182 1154	1004 1095 1045 1072	1097 1079 1089 1082	1053 1069 1059 1066	1224 1134 1183 1156	1007 1096 1047 1075	1101 1079 1091 1085
Centre of swing . . .	1060·80	1163·75	1062·60	1085·20	1062·95	1165·70	1064·65	1086·90
Deflection due to rider or mass	..	102·050	..	22·425	..	101·900	..	22·000
Mass deflection ÷ rider deflection	..	·221583	..	·219907	..	·217983	..	·215898

TABLE III. (continued).

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings	1054 1072 1061 1068	1226 1135 1185 1157	1009 1096 1048 1076	1101 1081 1093 1086	1054 1073 1064 1068	1225 1135 1187 1159	1008 1099 1048 1080	1102 1081 1094 1089
Centre of swing . . .	1065·15	1167·15	1065·35	1088·55	1066·85	1168·40	1067·00	1089·70
Deflection due to rider or mass	. .	101·90	. .	22·450	. .	101·475	. .	21·625
Mass deflection ÷ rider deflection	. .	·218106	. .	·220774	. .	·217172	. .	·213607

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings	1058 1075 1066 1072	1228 1138 1189 1161	1014 1102 1053 1080	1104 1086 1097 1091	1059 1076 1068 1074	1229 1141 1192 1163	1019 1103 1055 1081
Centre of swing . . .	1069·15	1170·80	1070·45	1093·00	1070·95	1173·40	1072·15
Deflection due to rider or mass	. .	101·000	. .	22·300	. .	101·850	
Mass deflection ÷ rider deflection	. .	·217458	. .	·219867			

May 25, afternoon. Mean of 25 determinations of M/R = a ⎫ ·21701412
Attracted masses in upper position ⎭

Mean of 50 determinations, morning and afternoon, ·2157379.

SUMMARY of Set I.

February 4	a = ·2142212
May 25	a = ·2157379
Mean value of .	a = ·2149791
April 30	A = 1·010905
May 4	A = 1·011032
Mean value of .	A = 1·0109685

therefore

$$A - a = ·7959894.$$

TABLE III. (continued).

SET II.

All Attracting and Attracted Masses inverted and changed over, each to the other side. The Suspending Rods also reversed and Riders interchanged. The initial position always the higher reading on the scale.

I. ATTRACTED Masses in Lower Position. July 28, 1890, 8.10 to 9.43 P.M. Temperature: in Observing Room, 17°–16°·9; in Balance Room, 15°·4; Barometer, 747·6–748 millims. Weather fine and calm; wind W.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings	1099 1051 1081 1063	912 1007 951 985	1130 1034 1093 1057	917 1005 952 985	1126 1036 1091 1058	914 1008 951 986	1131 1035 1093 1057	922 1005 954 984
Centre of swing . . .	1069·65	971·95	1070·55	972·10	1070·20	972·60	1070·95	973·15
Deflection due to rider or mass	. .	98·150	. .	98·275	. .	97·975	. .	97·575
Mass deflection ÷ rider deflection	. .	. .	. .	1·00217	. .	·99949	. .	·99541

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	1128 1035 1092 1058	913 1010 951 987	1134 1035 1095 1058	924 1006 956 987	1130 1038 1094 1061	915 1013 953 989	1137 1034 1098 1060	919 1012 955 989
Centre of swing . . .	1070·50	973·30	1072·25	975·00	1073·05	975·65	1073·80	976·40
Deflection due to rider or mass	. .	98·075	. .	97·650	. .	97·775	. .	97·700
Mass deflection ÷ rider deflection	. .	·99528	. .	·99719	. .	99898	. .	·99719

TABLE III. (continued).

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings	1132 1040 1095 1062	917 1014 954 989	1140 1036 1099 1060	924 1009 958 990	1134 1042 1098 1064	916 1016 955 993	1136 1041 1098 1064	960 991 972 983
Centre of swing . . .	1074·40	976·55	1075·05	977·40	1076·90	978·20	1076·70	979·05
Deflection due to rider or mass	..	98·175	..	98·575	..	98·600	..	98·100
Mass deflection ÷ rider deflection	..	·99962	..	1·00191	..	·99734	..	·99506

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings . . .	1133 1044 1098 1065	916 1018 956 994	1142 1039 1103 1064	925 1013 960 994	1134 1045 1101 1068	918 1019 957 997	1143 1042 1103 1066	925 1018 961 996
Centre of swing . . .	1077·60	979·55	1078·65	980·30	1079·80	981·00	1080·00	982·65
Deflection due to rider or mass	..	98·575	..	98·925	..	98·900	..	97·075
Mass deflection ÷ rider deflection	..	·99937	..	1·00190	..	·99090	..	·99031

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings . . .	1136 1046 1099 1068	924 1019 961 996	1145 1042 1104 1067	930 1016 964 995	1140 1046 1104 1069	918 1022 959 997	1143 1045 1104 1068	928 1018 962 996
Centre of swing . . .	1079·45	982·95	1080·75	983·50	1082·00	982·80	1081·80	983·30
Deflection due to rider or mass	..	97·150	..	97·875	..	99·100	..	98·875
Mass deflection ÷ rider deflection	..	1·00335	..	·99745	..	·99268	..	1·00051

TABLE III. (continued).

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings . . .	1138 1047 1104 1071	926 1020 963 997	1146 1045 1107 1069	928 1022 964 998	1142 1047 1106 1071	927 1021 963 999	1144 1047 1107 1070	937 1015 967 996
Centre of swing . . .	1082·55	984·45	1083·45	985·75	1083·70	985·10	1084·05	985·10
Deflection due to rider or mass	. .	98·550	. .	97·825	. .	98·775	. .	98·900
Mass deflection ÷ rider deflection	. .	·99797	. .	·99151	. .	·99582	. .	1·00139

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings . . .	1140 1049 1105 1072	923 1024 962 999	1148 1045 1108 1071	932 1021 966 998	1141 1050 1106 1072	924 1024 963 1001	1144 1048 1107 1072
Centre of swing . . .	1083·95	985·40	1084·35	986·60	1084·80	986·25	1084·75
Deflection due to rider or mass	. .	98·750	. .	97·975	. .	98·525	
Mass deflection ÷ rider deflection	. .	·99684	. .	·99328			

July 28, 1890. Mean of 25 determinations of M/R = A, Attracted masses in lower position } ·9973168.

TABLE III. (continued).

SEPTEMBER 17, 1890, 8.0 to 9.31 P.M. Temperature: in Observing Room, 17°–17°·5; in Balance Room, 15°·8. Barometer, 746·2–746·4 millims. Weather warm, cloudy.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings	1085 1051 1073 1058	908 1004 945 981	1118 1029 1085 1050	921 995 949 978	1109 1036 1081 1053	905 1006 944 981	1126 1026 1087 1050	921 996 951 978
Centre of swing . . .	1064·20	967·35	106·34	966·70	1063·75	967·35	1063·95	967·90
Deflection due to rider or mass	..	96·450	..	96·875	..	96·500	..	96·450
Mass deflection ÷ rider deflection	..	..	..	1·00415	..	1·00168	..	·99613

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	1113 1034 1084 1053	907 1006 944 982	1126 1027 1088 1052	929 993 953 978	1110 1038 1083 1056	910 1007 947 984	1131 1027 1092 1052	934 993 956 979
Centre of swing . . .	1064·75	967·75	1065·05	968·40	1065·90	969·90	1067·10	970·20
Deflection due to rider or mass	..	97·150	..	97·075	..	96·600	..	96·850
Mass deflection ÷ rider deflection	..	·99601	..	1·00206	..	1·00375	..	1·00026

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings	1104 1044 1081 1059	910 1008 947 985	1129 1030 1091 1054	924 1001 953 983	1116 1040 1086 1057	909 1009 947 986	1121 1036 1088 1056	927 1000 956 984
Centre of swing . . .	1067·00	970·44	1067·90	971·45	1066·70	970·85	1067·05	972·80
Deflection due to rider or mass	..	97·050	..	958·50	..	96·025	..	95·550
Mass deflection ÷ rider deflection	..	·99279	..	·99288	..	·99662	..	·99105

TABLE III. (continued).

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings	1112 1043 1086 1060	914 1009 951 987	1131 1033 1093 1056	929 1002 957 984	1114 1044 1087 1061	916 1011 952 988	1132 1035 1094 1058	934 1002 960 986
Centre of swing . . .	1069·65	973·10	1070·15	974·00	1070·70	974·50	1071·70	976·00
Deflection due to rider or mass	..	96·800	..	96·425	..	96·700	..	96·550
Mass deflection ÷ rider deflection	..	99161	..	·99664	..	·99780	..	·99690

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings	1097 1058 1083 1067	916 1015 954 991	1135 1037 1098 1061	942 999 965 986	1119 1048 1093 1066	919 1017 957 993	1136 1039 1099 1063	935 1006 963 989
Centre of swing . . .	1073·40	977·10	1074·80	977·85	1075·80	979·70	1076·25	979·10
Deflection due to rider or mass	..	97·000	..	97·450	..	96·325	..	97·025
Mass deflection ÷ rider deflection	..	1·00000	..	1·00815	..	1·00947	..	1·00362

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings	1122 1048 1093 1065	917 1018 956 994	1141 1038 1101 1062	929 1011 962 993	1118 1053 1093 1068	921 1019 958 996	1141 1041 1103 1065	941 1009 966 993
Centre of swing . . .	1076·00	979·45	1076·95	980·65	1078·20	981·40	1079·40	982·65
Deflection due to rider or mass	..	97·025	..	96·925	..	97·400	..	96·975
Mass deflection ÷ rider deflection	..	·99948	..	·99704	..	·99538	..	·99628

TABLE III. (continued).

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings	1134 1047 1100 1067	920 1022 958 998	1143 1041 1104 1065	932 1016 964 996	1134 1048 1102 1068	925 1021 962 997	1140 1045 1104 1068
Centre of swing . . .	1079·85	982·65	1080·00	983·85	1081·15	984·20	1081·55
Deflection due to rider or mass	..	97·275	..	96·725	..	97·150	
Mass deflection ÷ rider or mass	..	·99563	..	·99499			

September 1890. Mean of 25 determinations of M/R = A } ·9984148.
Attracted masses in lower position }

July 28 and September 17. Mean of 50 determinations of M/R = A, ·9978658.

II. ATTRACTED Masses in Upper Position. September 23, 1890, 7.52 to 9.30 P.M. Temperature: in Observing Room, 15°·3–15°·4; in Balance Room, 15°·05. Barometer, 749·8–750·2 millims. Weather, light S.W. wind and clear after heavy showers. Scale readings between about 1100 and 1300; 1000 omitted.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings	307 248 285 263	113 210 151 186	329 235 293 257	235 257 243 251	281 261 273 265	112 208 149 185	326 232 290 256	233 256 241 249
Centre of swing . . .	271·00	173·10	270·95	248·25	268·35	171·45	268·25	246·60
Deflection due to rider or mass	..	97·875	..	21·400	..	96·850	..	21·175
Mass deflection ÷ rider deflection.	..	..	..	·219797	..	·219799	..	·218581

TABLE III. (continued).

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	279 260 272 264	110 207 148 183	331 228 290 253	232 255 239 248	277 258 271 262	110 205 147 182	324 229 288 252	230 254 239 247
Centre of swing . . .	267·30	170·15	266·80	245·15	265·80	168·90	265·55	244·50
Deflection due to rider or mass	..	96·900	..	21·150	..	96·775	..	20·225
Mass deflection ÷ rider deflection	..	·218395	..	·218407	..	·213769	..	·209179

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings	275 256 269 261	108 204 145 181	323 228 286 251	228 253 237 247	276 255 268 260	107 203 145 179	328 224 287 249	226 252 236 245
Centre of swing . . .	263·90	167·40	264·10	243·15	263·00	166·65	263·25	241·85
Deflection due to rider or mass	..	96·600	..	20·400	..	96·475	..	20·225
Mass deflection ÷ rider deflection	..	·210274	..	·211317	..	·210547	..	·209652

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings	274 254 265 258	106 199 143 176	320 224 283 246	232 245 237 241	271 252 262 255	100 197 138 175	317 221 281 243	222 247 232 241
Centre of swing . . .	260·90	163·90	260·40	239·85	258·25	160·50	257·90	237·60
Deflection due to rider or mass	..	96·750	..	19·475	..	97·575	..	19·100
Mass deflection ÷ rider deflection	..	·205323	..	·200437	..	·197668	..	·196730

TABLE III. (continued).

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings	266 249 259 254	98 197 136 173	317 219 279 241	221 244 228 237	265 246 257 251	97 193 135 171	311 218 275 240	218 242 226 235
Centre of swing . . .	255·50	159·10	255·90	234·20	253·05	157·00	253·30	232·10
Deflection due to rider or mass	..	96·600	..	20·275	..	96·175	..	20·150
Mass deflection ÷ rider deflection	..	·203804	..	·210351	..	·210164	..	·209271

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings	264 243 256 249	95 191 133 168	310 216 273 238	215 239 223 232	261 241 253 246	91 188 128 166	311 210 272 234	212 236 220 229
Centre of swing . . .	251·20	154·90	251·40	229·10	248·55	151·10	248·40	226·10
Deflection due to rider or mass	..	96·400	..	20·875	..	97·375	..	20·675
Mass deflection ÷ rider deflection	..	·212785	..	·215456	..	·213350	..	·213585

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)	*m.* (56)
Scale readings	257 237 250 243	90 186 127 162	306 208 269 232	211 234 218 227	256 236 248 242	88 184 125 160	303 209 265 231	208 232 216 225
Centre of swing . . .	245·15	149·25	245·80	224·10	244·25	147·21	243·95	222·10
Deflection due to rider or mass	..	96·225	..	20·925	..	96·900	..	20·350
Mass deflection ÷ rider deflection	..	·216160	..	·216699	..	·212977	..	·209956

TABLE III. (continued).

	i. (57)	r. (58)	i. (59)	m. (60)	i. (61)
Scale readings	253 233 246 238	86 180 122 157	301 205 263 227	203 228 213 222	293 204 280* 245
Centre of swing . . .	240·95	144·00	240·95		
Deflection due to rider or mass	. .	96·950			

September 23, 1890. Mean of 27 determinations of M/R = a } ·2112753.
Attracted masses in upper position }

SEPTEMBER 25, 1890, 7.10–8.43 P.M. Temperature: in Observing Room, 15°–15°·2; in Balance Room, 15°. Barometer, 760·8, steady. Weather cloudy, with westerly airs. Time of swing 21 seconds. 1000 omitted in scale readings.

	i. (1)	r. (2)	i. (3)	m. (4)	i. (5)	r. (6)	i. (7)	m. (8)
Scale readings	246 238 243 239	84 179 121 156	301 205 263 228	206 229 215 224	248 233 243 236	82 178 119 156	297 204 260 226	202 228 212 222
Centre of swing . . .	240·90	142·90	240·95	220·40	248·95	141·60	238·95	218·10
Deflection due to rider or mass	. .	98·025	. .	19·550	. .	97·350	. .	20·850
Mass deflection ÷ rider deflection	. .	. .	. .	·200128	. .	·207499	. .	·213163

* This is a considerable rise, showing either a sudden disturbance or a displacement of the apparatus; possibly the telescope was touched. The rise was maintained and therefore the observations were discontinued.

TABLE III. (continued).

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings	248 233 243 236	83 176 119 155	300 203 261 226	204 228 214 224	252 233 245 239	84 182 122 158	303 206 265 228	207 232 217 226
Centre of swing . . .	238·95	140·80	239·20	219·50	240·70	144·60	242·45	222·60
Deflection due to rider or mass	..	98·275	..	20·450	..	96·975	..	20·825
Mass deflection ÷ rider deflection	..	·210125	..	·209475	..	·212813	..	·214718

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings	255 237 249 242	87 184 125 162	307 207 267 232	210 234 219 228	257 238 251 244	90 184 127 163	271 233 255 241	215 233 222 229
Centre of swing . . .	244·40	147·55	244·70	224·65	246·05	148·75	246·70	226·15
Deflection due to rider or mass	..	97·000	..	20·725	..	97·625	..	20·725
Mass deflection ÷ rider deflection	..	·214175	..	·212974	..	·212292	..	·212564

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings	258 241 251 244	90 186 129 164	307 213 270 236	213 237 223 232	262 242 253 246	93 189 131 167	307 215 272 237	215 239 225 233
Centre of swing . . .	247·05	150·45	248·60	228·30	248·85	153·00	250·25	230·05
Deflection due to rider or mass	..	97·375	..	20·425	..	96·550	..	19·750
Mass deflection ÷ rider deflection	..	·211297	..	·210649	..	·208053	..	·204425

TABLE III. (continued).

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings	261 243 253 247	93 189 132 167	312 214 273 237	215 241 225 235	263 245 257 250	96 192 135 168	312 217 275 240	217 243 227 237
Centre of swing . . .	249·35	153·40	250·80	231·10	252·40	156·05	253·05	233·05
Deflection due to rider or mass	..	96·675	..	20·500	..	96·675	..	20·525
Mass deflection ÷ rider deflection	..	·208172	..	·212051	..	·212180	..	·212254

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings	264 247 259 251	98 194 136 171	314 219 277 242	220 243 231 238	267 249 260 255	100 197 138 174	321 223 281 246	224 246 233 242
Centre of swing . . .	254·10	157·85	255·05	235·20	256·25	160·35	259·30	238·05
Deflection due to rider or mass	..	96·725	..	20·450	..	97·425	..	21·250
Mass deflection ÷ rider deflection	..	·211812	..	·210662	..	·214011	..	·218650

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings	271 252 264 256	102 200 139 176	321 221 282 245	224 247 233 242	271 251 264 256	102 198 140 174	314 226 280 247
Centre of swing . . .	259·30	162·20	259·00	238·40	258·95	161·60	259·50
Deflection due to rider or mass	..	96·950	..	20·575	..	97·625	
Mass deflection ÷ rider deflection	..	·215704	..	·211487			

September 25, 1890. Mean of 25 determinations of $M/R = a$, Attracted masses in upper position } ·21125332.

September 23 and September 25 } Mean of 52 determinations of $M/R = a$, ·2112647.

TABLE III. (continued).

SUMMARY of Set II.

July 28	$A = \cdot 9973168$
September 17	$A = \cdot 9984148$
Mean value of .	$A = \cdot 9978658$
September 23	$a = \cdot 2112753$
,, 25	$a = \cdot 2112533$
Mean value of .	$a = \cdot 2112647$

Therefore

$$A - a = \cdot 7866011.$$

Mean value, giving equal weights to Sets I. and II.,

$$A - a = \cdot 791295.$$

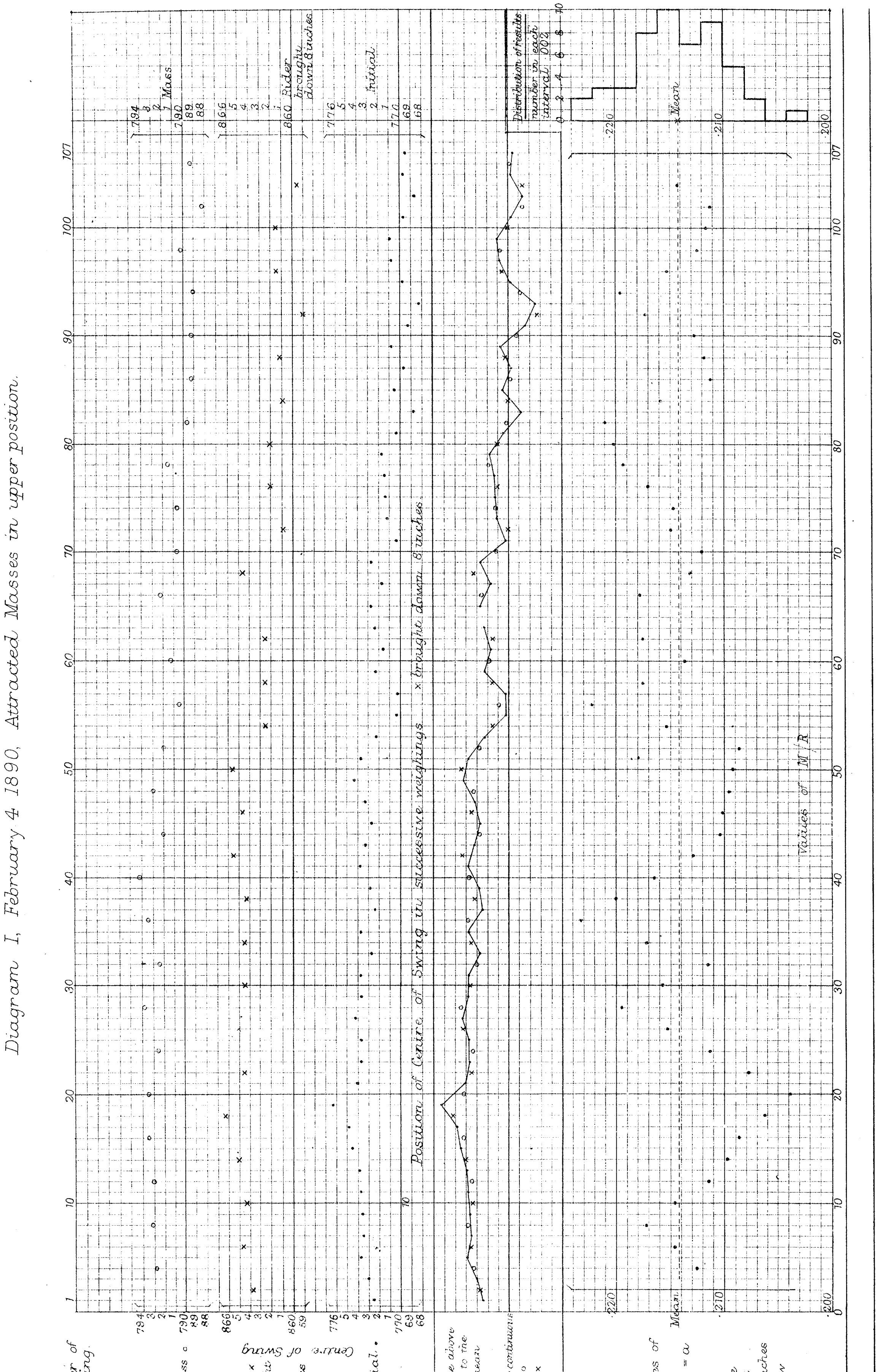
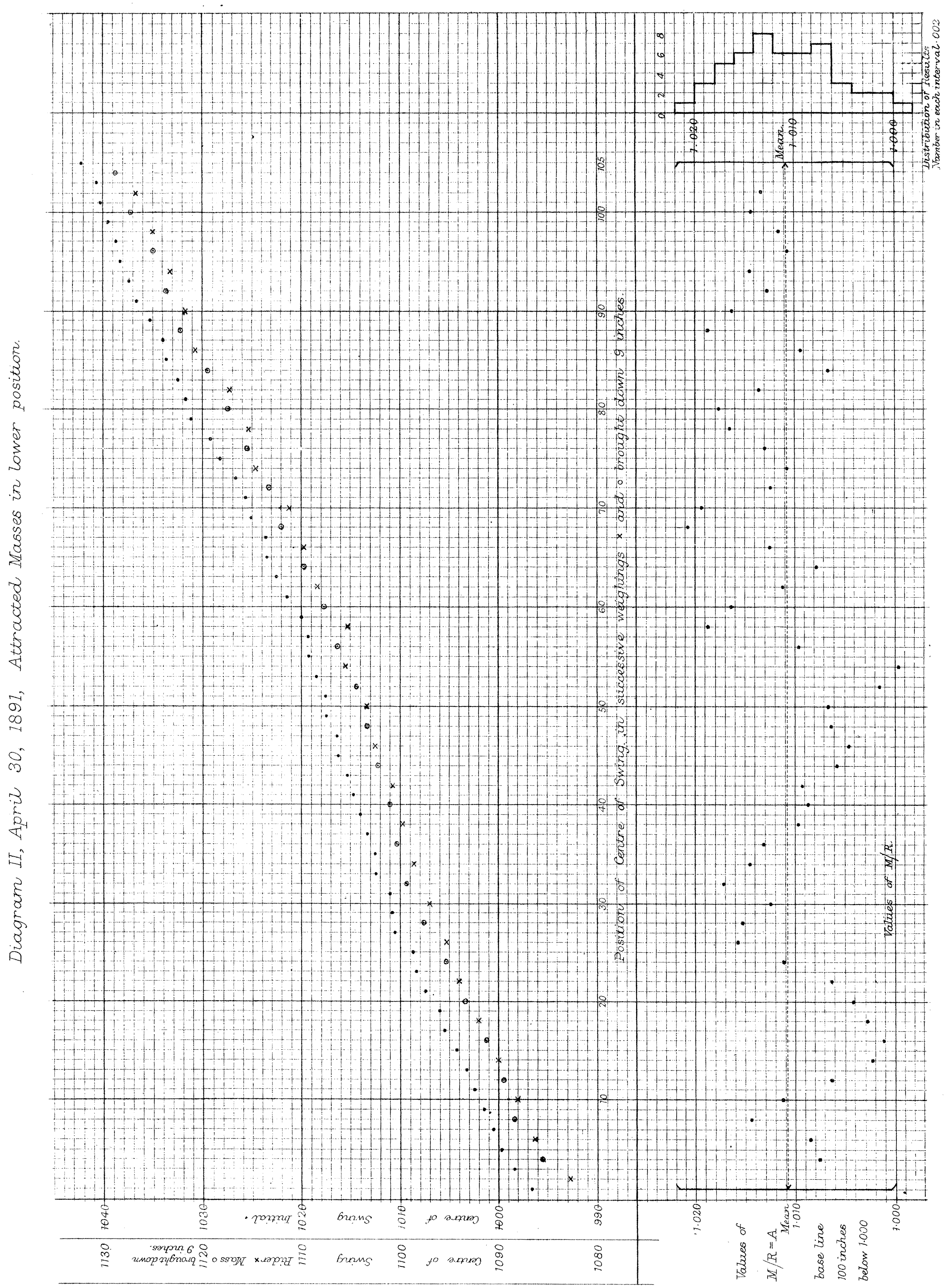


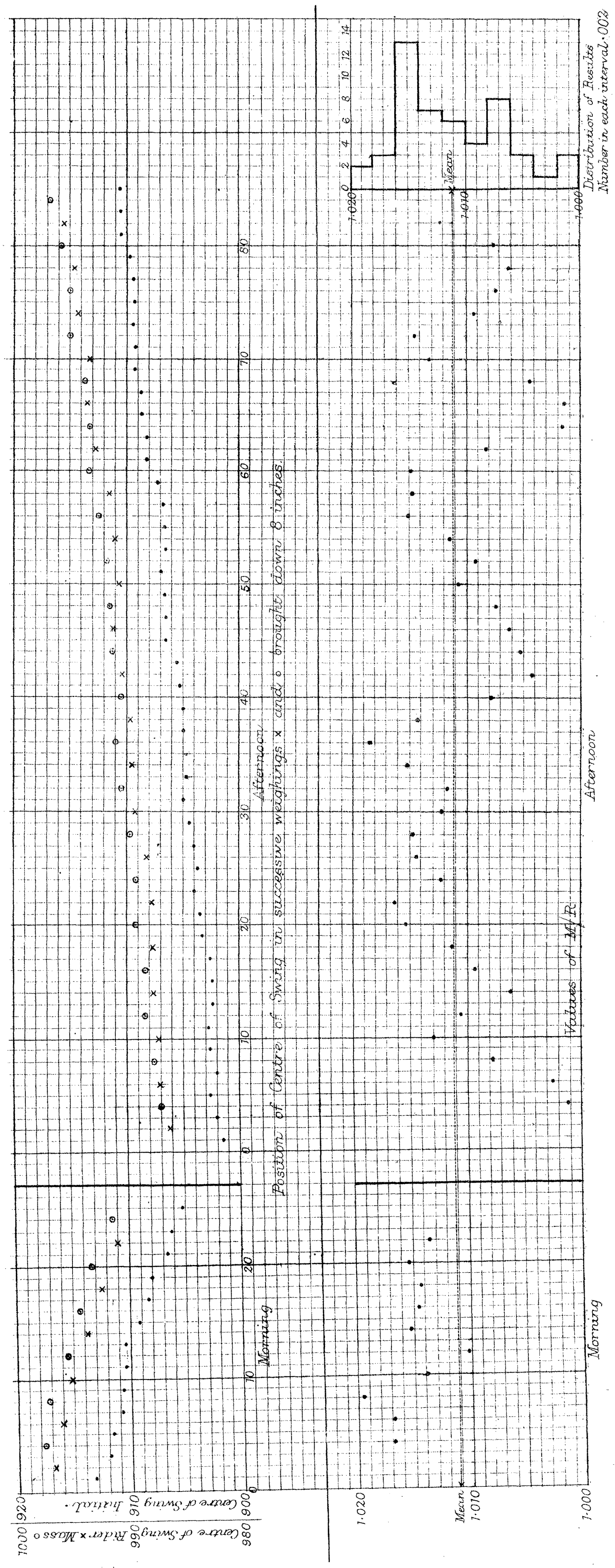
Diagram I, February 4 1890, Attracted Masses in upper position.

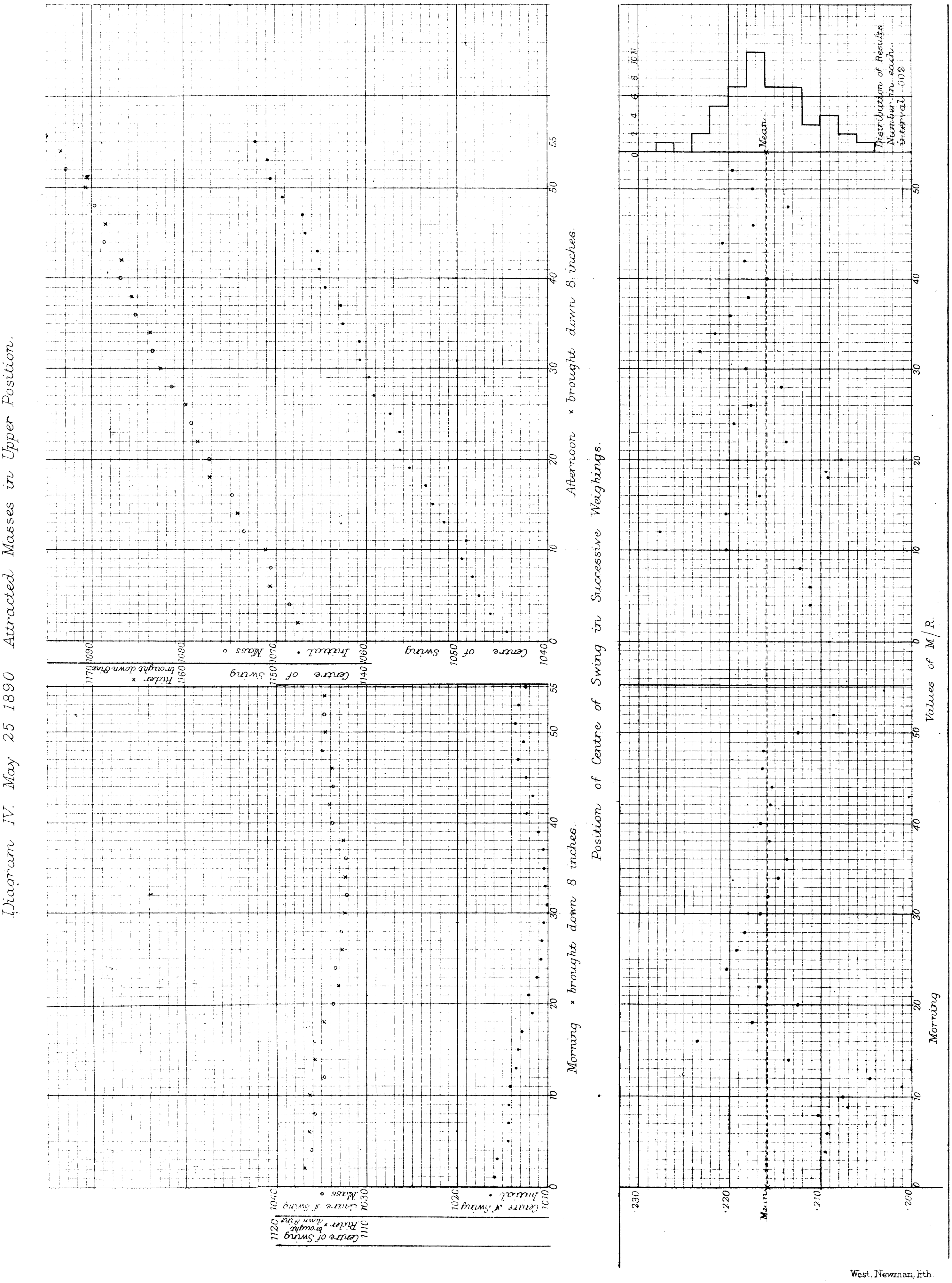
West, Newman, lith.



West, Newman, lith.

Diagram III, May 4, 1890, Attracted Masses in lower position.





West, Newman, lith.

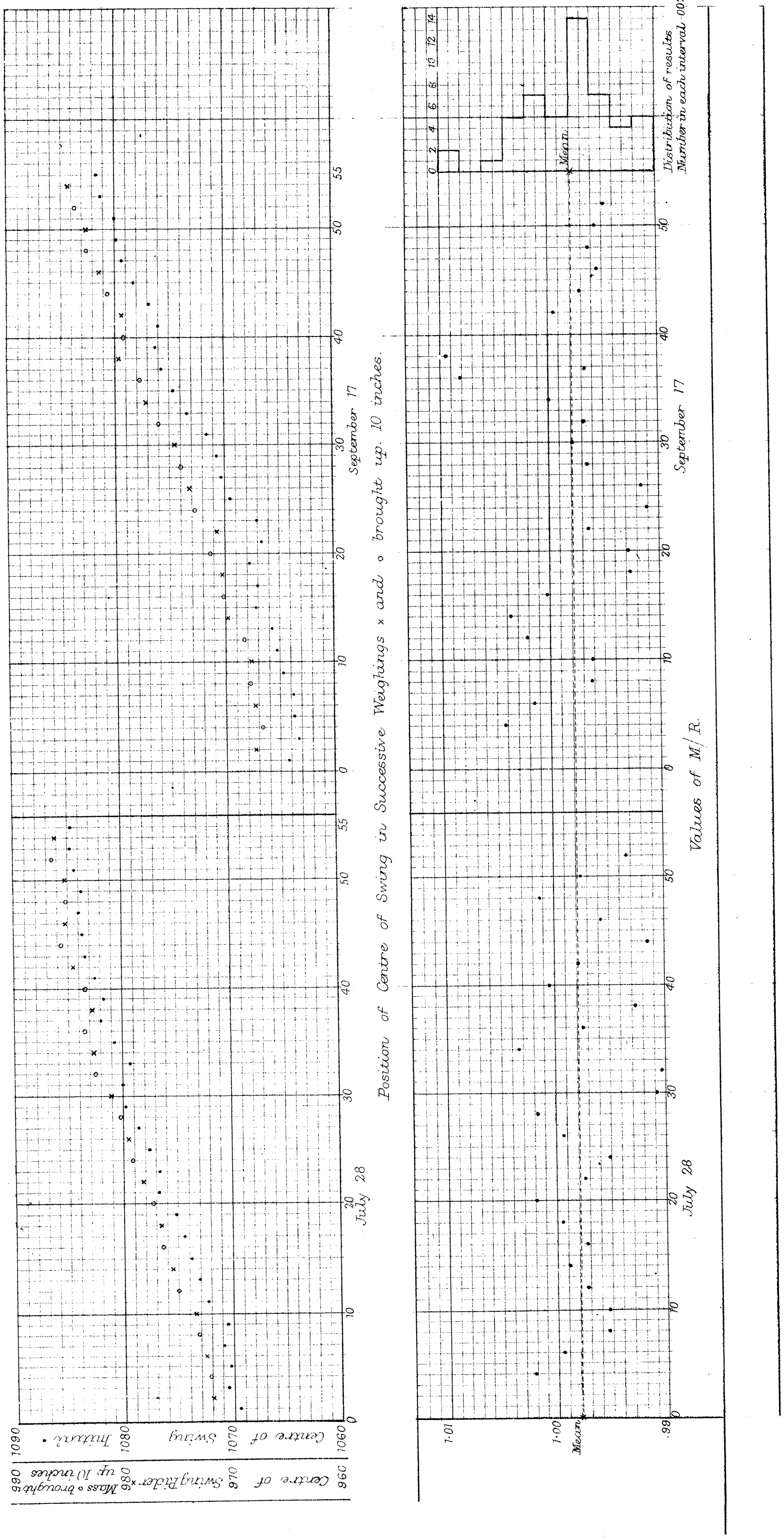
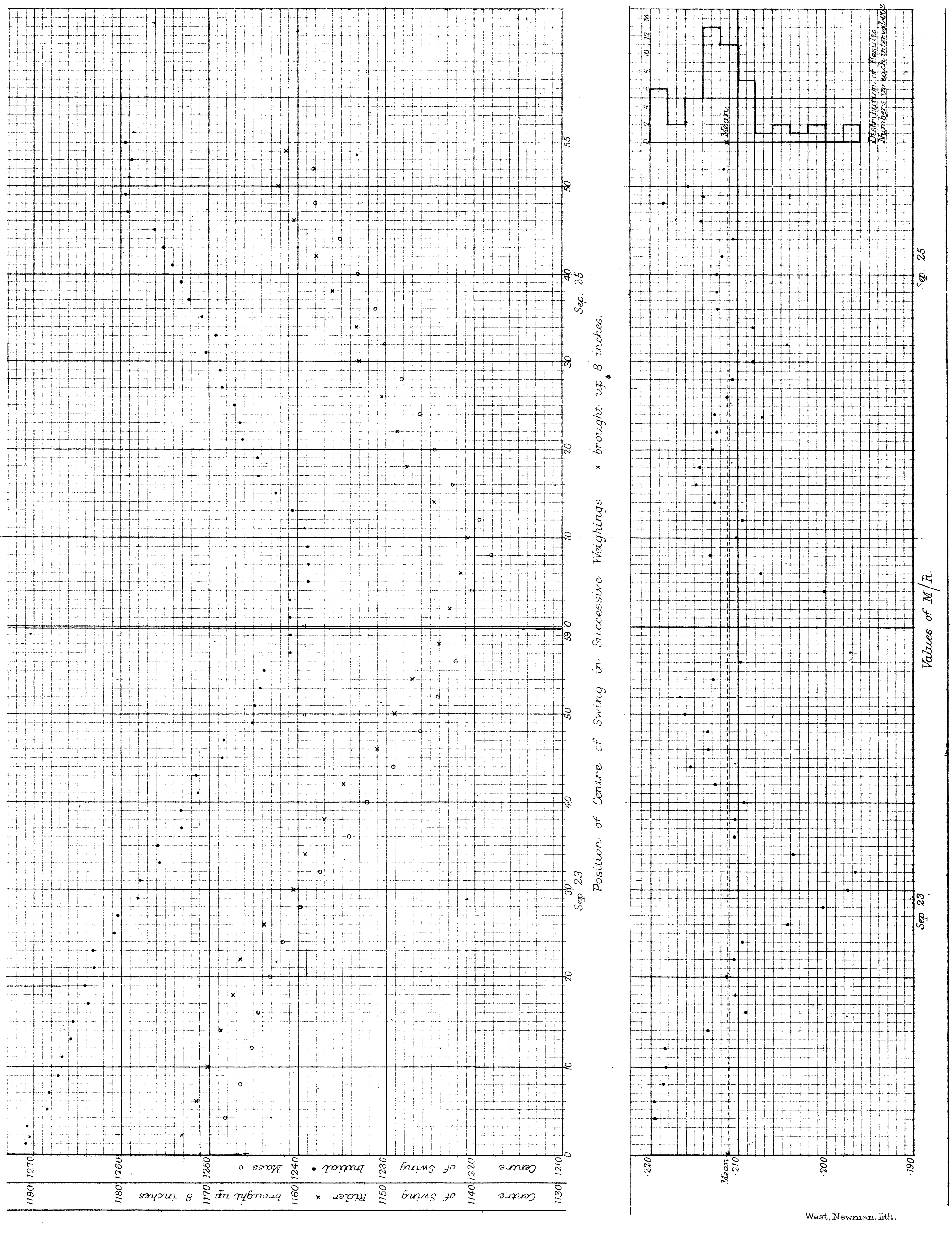


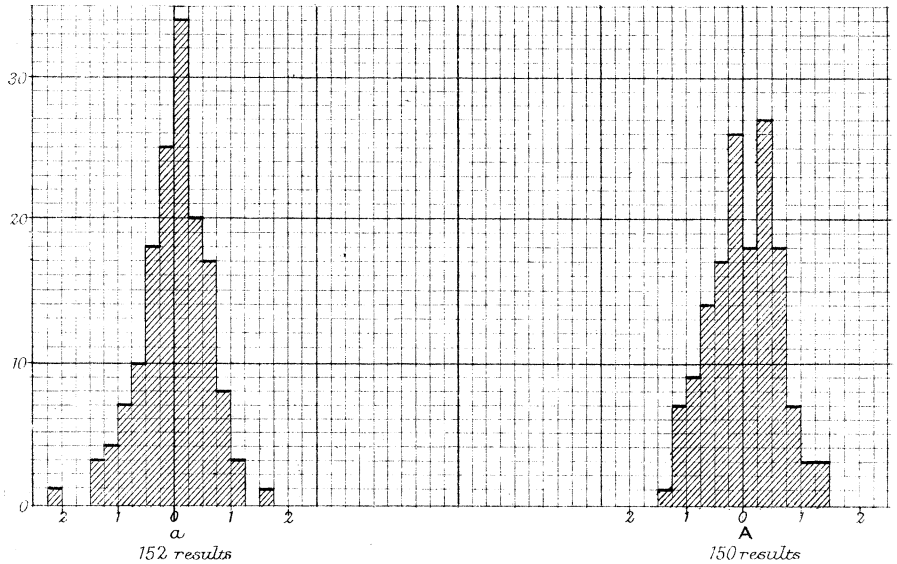
Diagram V. July 28 and Sep. 17. Attracted Masses in Lower Position. Distances nearly equal, on the two dates.

West, Newman. lith.



West, Newman, lith.

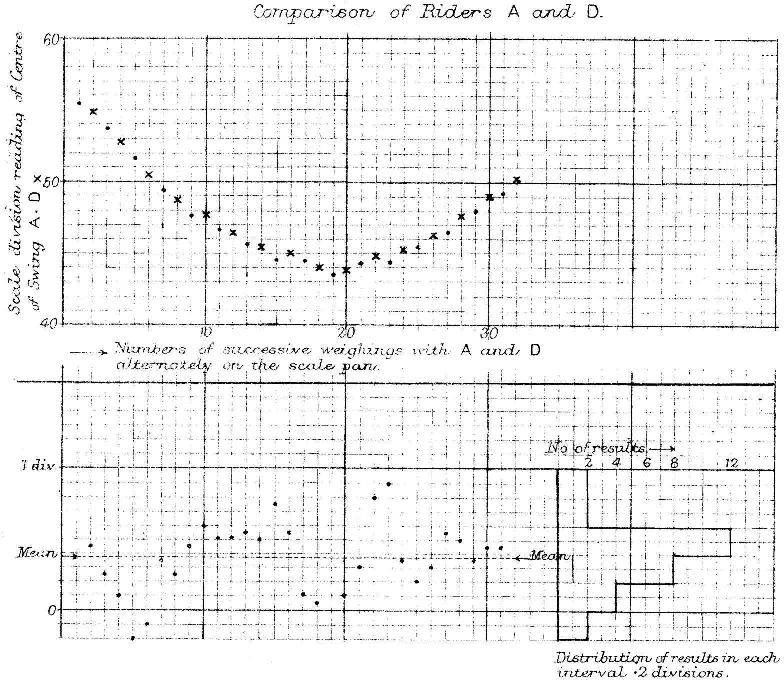
Diagram VII.



Distribution of results about the means assumed correct for each set of distances. The numbers along the base are in percentages of the distance a A. The numbers on the side line show the numbers in each interval 0·25 per cent from the mean. The distance a A should be 40 inches to show both on the same diagram.

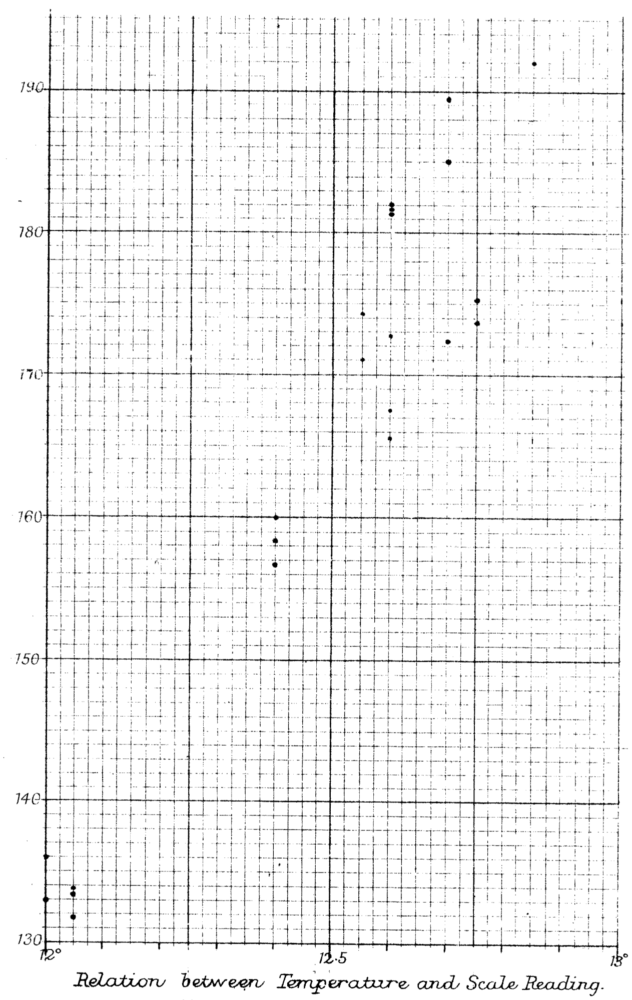
Diagram VIII.

Comparison of Riders A and D.



Successive values of D – A in terms of 1 scale divion = ·0035 mgm.

Diagram IX.



Relation between Temperature and Scale Reading. May 9–22.

West, Newman, lith.